Introduction to Uncertainty

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One can never know how well he knows a thing, only how strongly he believes it.

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1 Correspondence should be directed to tvosk@comcast.net. See also, Vosk, Measurement Uncertainty: Forensic Metrology and Result Interpretation Part I & II in Understanding DUI Scientific Evidence, Chapters 7 & 8 (4th ed. Aspatore, 2011); Vosk & Emery, Forensic Metrology: A Primer on Scientific Measurement for Lawyers, Judges and Forensic Scientists (CRC Press – In Preparation).
For Kris: Who believed in me when no one else did and made all things possible.
INTRODUCTION

Revealed Truth, absolute and known, is not the domain of science. Rather, it is relative inference. From observation and data, to the relationships alive therein, to varying degrees of certitude never complete. That’s the promise of science... and the best it can do.

Exactly what constitutes “science” has long been a matter of debate. The word “science” comes from the Latin scientia, meaning knowledge. This is natural enough as science is characterized by the goal of acquiring knowledge. There are many activities where the goal is the acquisition of knowledge, however, that would not be seen as science. For example, both philosophy and religion are frequently relied upon as a source of, or tool for obtaining, knowledge, but neither constitutes science. So, what is it that separates these disciplines from what would be characterized as science?

Some consider falsifiability and testability to be the hallmarks of science. The object here is the contention being made. A contention is scientific if it is open to being empirically tested and falsified. Others see science not so much as centering on the status of particular propositions, but instead as more of a puzzle solving activity. Here, the focus is on the actions engaged in. Actions are scientific if they are aimed at empirically solving unanswered questions. Still others claim that science is characterized by the ability to successfully predict future phenomenon. This looks at forecasting. Forecasts are scientific if they are empirically based and able to predict unknown outcomes with a high degree of certainty.

There is nothing necessarily incompatible about these different conceptions. In their less dogmatic forms, each seems to capture some aspect of the scientific enterprise. Like the three blind men who, upon touching different parts of an elephant described very different beasts, these descriptions of science seem to convey distinct characteristics of a greater whole. Which
aspect one focuses on may simply be a function of whether we are considering a body of accumulated knowledge or instead the practices engaged in to obtain such knowledge.

One thing common to each of these frameworks is that each has an empirical basis. Whatever science is, all agree that it requires collection and analysis of information from the physical world in order to gain knowledge about the physical world. It “is based on the principle that…observation is the ultimate and final judge of the truth.”\(^1\) For any claim to be considered scientific, it must either be derived from and/or withstand the test of, systematic observation. Thus, unlike religion or philosophy which may or may not choose to incorporate rigorously obtained empirical information, science requires it.

Another fundamental aspect of science is that in addressing “truth”, science does not deal in absolutes.

Ontologically, although science must be empirically based, its true subject matter, as a human enterprise, is simply information and the relationships inherent within it. Although we ascribe an external reality to revealed physical principles and/or conclusions, in the end we are limited by our evolved senses and manner of reason. And while these tools may serve us well in an evolutionary sense, only faith connects their products to fundamental reality. In the end, science deals strictly with information, the manner in which it is processed and the relationships found or created within it by inference. Our scientific “truths” consist of little more than models of information networks that rigorously correspond to experience, which may or may not correspond to deeper reality.

Epistemologically, when science is relied upon to guide our understanding of the physical world, the absolute is still denied to us by the fact that uncertainty is inherent to all scientific knowledge. The relationships perceived and the conclusions drawn there-from are all soft edged to
Although the degree of belief associated with a given proposition may be high, science cannot prove it. “It is scientific only to say what is more likely and what is less likely.” Thus, even when the connection between experience and physical reality is accepted, the best we can do is speak of the relative strength of our inferences.

In this context, however, one of the remarkable aspects of science is its ability to rigorously characterize the relative strength of our inferences and communicate the level of uncertainty associated with certain types of claims. That is, we can grade the degree of fuzziness surrounding certain scientific claims so that their strengths can be widely understood and compared. This is critical to our understanding of science or any claim made in its name. In this manner, science brings order to our world through the creation and utilization of models which enable us to make predictions and/or inferences with relative degrees of certainty concerning the physical phenomena of our experience.

From this brief discussion, a simple picture of what constitutes science can be painted. Science focuses on the quest for, and acquisition of, knowledge of the physical world as it is perceived and processed by our senses. It requires the systematic collection of empirical information followed by an assessment of the strengths and weaknesses of that information. Where the information permits, relationships are discovered and causal inferences are made creating knowledge in the form of an explanation for what has been observed and the ability to determine what will happen in the future when certain criteria are satisfied. Finally, based on our information, the significance of that knowledge is evaluated through a rigorous determination of its limits and, in particular, the degree of certainty associated with it. Absent a rigorous understanding of these limits, however, the weight accorded scientific knowledge/conclusions is a matter of faith. Science requires more.
The uncertainty associated with a piece of scientific knowledge provides a measure of its practical epistemological robustness. It represents the degree to which our knowledge concerning the relevant physical phenomenon is imperfect. Although uncertainty may be minimized, it cannot be eliminated: it is inherent to all scientific knowledge. Fortunately, science has developed methods for rigorously characterizing and communicating the level of uncertainty associated with certain types of claims. These methods permit us to assign a “degree of belief” that can be placed in such knowledge expressed as a likelihood or level of confidence.

Consider the case of scientific measurement. To most individuals, the value obtained by a scientific measurement means exactly what it says. Thus, if the value reported by a breath test instrument is 0.08, most individuals will assume that the measurement represents a true and accurate value for an individual’s BrAC of 0.08.

For even the most carefully performed measurement, however, the value of a thing being measured (the “measurand”) can never be known exactly; all that can ever be given is an estimated value. In fact:

…for a given measurand and a given result of measurement of it, there is not one value but an infinite number of values dispersed about the result that are consistent with all of the observations and data and one’s knowledge of the
physical world, and that with varying degrees of credibility can be attributed to the measurand.

Thus, in the context of a breath test yielding a value of 0.08, the truth is more accurately represented as a packet of values, any of which may represent an individual’s actual BrAC.

In essence, “[t]he result of a measurement is a probability distribution that provides an unambiguous encoding of one’s state of knowledge about the measured quantity.” The uncertainty associated with a measurement supplies a quantitative statement characterizing the dispersion of values that can actually and “reasonably be attributed to the measurand.”

“Requirements for measurement accuracy translate into a need to know not only the results of measurements but the uncertainties associated with the results.” “The result of a measurement cannot be correctly evaluated without knowing its uncertainty.”

Knowledge of the uncertainty associated with measurement results is essential to the interpretation of the results. Without quantitative assessments of uncertainty, it is impossible to decide whether observed differences between results reflect more than experimental variability, whether test items comply with specifications, or whether laws based on limits have been broken. Without information on uncertainty, there is a risk of misinterpretation of results. Incorrect decisions taken on such a basis may result in unnecessary expenditure in industry, incorrect prosecution in law, or adverse health or social consequences.
Accordingly, “[a] result is complete only when accompanied by a quantitative statement of its uncertainty.” The most common way of expressing measurement uncertainty is as a coverage interval. It consists of a best estimate of the “true” value of the measurand accompanied by a range of values that can also be attributed to the “true” value with a given level of confidence (probability).

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<th>Measurement Result</th>
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<tr>
<td>Measured mean value: 22 mg/dL</td>
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<tr>
<td>Uncertainty (k=2; 95%): ±2.5 mg/dL</td>
</tr>
<tr>
<td>Result: 22 ± 2.5 mg/dL (95%)</td>
</tr>
</tbody>
</table>

When the result of a forensic measurement is reported simply as “‘a number,’ it does not reflect the accuracy of the measurement and cannot be properly interpreted.” For example, methods for measuring the level of blood alcohol in an individual…can do so only within a confidence interval of possible values. “Estimating and reporting measurement uncertainty with the number completes the picture and allows us to properly use the result to make reliable and defensible decisions.” The importance of uncertainty as part of a complete result can be illustrated using a simple example provided by the National Academy of Sciences.

Consider…a case in which an instrument (e.g., a breathalyzer such as Intoxilyzer) is used to measure the blood-alcohol level of an individual three times, and the three measurements are 0.08 percent, 0.09 percent, and 0.10 percent. The variability in the three measurements may arise from the internal components of the instrument, the different times and ways in which the measurements were taken, or a variety of other factors. These measured results need to be reported, along with a confidence interval that has a high probability of containing the true blood-alcohol level (e.g., the mean plus or minus two standard deviations). For this illustration, the average is 0.09 percent and the standard deviation is 0.01 percent; therefore, a two-standard-deviation confidence interval (0.07 percent, 0.11 percent) has a high probability of containing the person’s true blood-alcohol
level. (Statistical models dictate the methods for generating such intervals in other circumstances so that they have a high probability of containing the true result.)

In this example, each of the measured values equals or exceeds a BrAC of 0.08, the level at which most states have defined the crime of *per se* DUI. If a jury is supplied with only the values reported by the instrument, the picture created overwhelming suggests, and is likely to result in, guilt.

Unfortunately, this picture is both incomplete and misleading. Once the uncertainty is included, we see that values as low as 0.07 may actually and reasonably be attributed to this individual’s true BrAC.
Of course, the uncertainty here does not dictate a particular result. It simply provides the decision maker with a complete and honest picture of the scientific “truth” so that all the evidence can be properly understood and weighed. Without more, though, it is obvious that an average juror may discover reasonable doubt hiding in the previously unreported uncertainty associated with these results. Clearly, “considering or not the uncertainty of a critical result can make the difference between acquittal and a guilty sentence.”

The same general principles apply to scientific calculations based upon measured or experimentally determined values. Even where the relationship between such physical quantities is well understood, the values assigned to each component quantity are typically accompanied by uncertainty which translates into uncertainty associated with the final calculated result. If the results of such calculations are to be interpretable, then the uncertainty associated with each must be determined and communicated to those who intend to rely upon them.
A BRIEF TOUR

ILLUSIONS OF CERTAINTY

“It is scientific only to say what is more likely and what is less likely.”

ACCURATE AND RELIABLE

“When uttered by an expert witness, the phrase ‘accurate and reliable’ is infused with talismanic connotation, beckoning one to trust the result presented. Unfortunately, it conveys little real information concerning how good a result actually is or what it means.”

“Because ‘accuracy’ is a qualitative concept, one should not use it quantitatively, that is, associate numbers with it.”
**Measurement Error**

The objective of error analysis is to determine an estimate of a measurand’s value that is as close as possible to the true value by identifying, accounting for and minimizing as many sources of measurement error as possible.\(^{20}\)

The Model

\[ Y = \bar{y} - \varepsilon \]

where

\( \bar{y} = \text{mean of set of measurements} \)
\( \varepsilon = \text{measurement error} \)

Two Types of Error: Systematic and Random

![Systematic Error & Bias Diagram]

\[ b_{tas} = Y - R \]
Correction Constant Bias
\[ Y_c = \bar{y} - b_c \]

Correction Percent Bias
\[ Y_c = \frac{\bar{y}}{1 + b_c} \]

Standard Deviation
\[ \sigma_y = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n - 1}} \]
The Model

\[ Y = \bar{y} - \varepsilon_T \]

Where

\[ \varepsilon_T = \varepsilon_{sys} + \varepsilon_{ran} \]
Frequentist theory of probability: Probability defined as relative frequency of occurrence over the universe (population) of possible events.

A Problem for Error Analysis

\[ \varepsilon = \varepsilon_{sys} \left[ \begin{align*} \frac{\varepsilon_{ran}}{\varepsilon} \end{align*} \right] \]

Measurement “error is an unknowable quantity in the realm of the state of nature.”

\(^{21}\)}
MEASUREMENT UNCERTAINTY

Error and Uncertainty are “not synonyms, but represent completely different concepts; they should not be confused with one another.”\textsuperscript{22}

“…for a given measurand and a given result of measurement of it, there is not one value but an infinite number of values dispersed about the result that are consistent with all of the observations and data and one’s knowledge of the physical world, and that with varying degrees of credibility can be attributed to the measurand.”\textsuperscript{23}

“The result of a measurement is a probability distribution that provides an unambiguous encoding of one’s state of knowledge about the measured quantity.”\textsuperscript{24}
The probability that a measurand’s value lies within a specified range of values can be visualized as the area under the curve spanning the range. The proportion of the area under the curve spanning our range of values to the total area under the curve yields the probability that the measurand’s value is contained within the specified region.

\[ P_{rob} = \]  

The expanded uncertainty, \( U \), defines “an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.” ²⁵
“Knowledge of the uncertainty associated with measurement results is essential to the interpretation of the results. Without quantitative assessments of uncertainty, it is impossible to decide...whether laws based on limits have been broken. Without information on uncertainty, there is a risk of misinterpretation of results. Incorrect decisions taken on such a basis may result in unnecessary expenditure in industry, incorrect prosecution in law, or adverse health or social consequences.”

A coverage interval is an “interval containing the set of true quantity values of a measurand with a stated probability, based on the information available.”
“In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement and thus the result is complete only when accompanied by a quantitative statement of its uncertainty.”

“Given the inherent variability of measurement, a statement of a measurement result is incomplete (perhaps even meaningless) without an accompanying statement of the estimated uncertainty of measurement.”

“When the value of a measurand is reported, the best estimate of its value and the best evaluation of the uncertainty of that estimate must be given.”

Measurement Result = Best Estimate ± Uncertainty

\[ Y = \bar{Y}_e \pm U \ (99\%) \]

Bayesian theory of probability: Probability defined as an information based degree of belief that an event will occur.

“Uncertainty explicitly acknowledges our lack of information and instead of claiming to tell us ‘what reality is’ it simply says that, based on the information that was considered, this is ‘what we believe reality to be.’ This is the essence of the uncertainty paradigm. It does not claim to convey the truth of a thing; it claims only to convey what is believed about a thing.”

“…uncertainty (in measurement) is a quantifiable parameter in the realm of the state of knowledge about nature.”
EXAMPLE

Identical test results, different meanings

--- BREATH ANALYSIS ---

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--- BREATH ANALYSIS ---

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BrAC Test 1

Test Values: .084, .081
Mean Meas. BrAC: .0825
Cov. Int.: .0749 - .0903

BrAC Test 2

Test Values: .084, .081
Mean Meas. BrAC: .0825
Cov. Int.: .0749 - .0903

BrAC Test 1

Test Values: .084, .081
Mean Meas. BrAC: .0825
Cov. Int.: .0749 - .0903

P < 0.08: ~ 19.3%

BrAC Test 2

Test Values: .084, .081
Mean Meas. BrAC: .0825
Cov. Int.: .0749 - .0903

P < 0.08: ~ 9.2%
Determining the Probability that \( \text{BrAC} < 0.08 \, \text{g/210L} \) in 6 Easy Steps

Step 1: Start with the coverage interval.

\[ b_l \leftrightarrow b_u \]

Step 2: Determine the bias corrected mean.

\[ \overline{\text{BrAC}}_c = \frac{(b_l + b_u)}{2} \]

Step 3: Determine the expanded uncertainty.

\[ U = \text{BrAC}_c - b_l \]

Step 4: Determine the combined uncertainty.

\[ \mu_c = \frac{U}{2.576} \]

Step 5: Determine the z-factor.

\[ z_{\text{BrAC}_c \rightarrow .08} = \frac{(\text{BrAC}_c - .08)}{\mu_c} \]

Step 6: Determine probability from statistical table.

\[ P(z_{\text{BrAC}_c \rightarrow .08})_{\text{BrAC} < .08} \]

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EXERCISE\textsuperscript{34}

BREATH ALCOHOL UNCERTAINTY: WASHINGTON STATE

--- BREATHE ANALYSIS ---

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Some necessary relationships

1. $BrAC = \overline{BrAC}_c \pm U \ (99\%)$

2. $\overline{BrAC}_c = \frac{BrAC}{(1+(b \cdot 0.01))}$

3. $\mu_c = \overline{BrAC}_c \sqrt{\left(\frac{QAP_{SD}/\sqrt{10}}{QAP_{mean}}\right)^2 + \left(\frac{\mu_{Qsol}}{Qsol_{ref}}\right)^2 + \left(\frac{(\mu_{b/s}/\sqrt{2})}{\overline{BrAC}_c}\right)^2}$

where

$\overline{BrAC}_c \equiv$ bias corrected mean
$QAP_{SD} \equiv$ QAP standard deviation
$QAP_{mean} \equiv$ QAP mean
$\mu_{Qsol} \equiv$ Standard uncertainty of QAP solution
$Qsol_{ref} \equiv$ Vapor concentration of QAP solution
$\mu_{b/s} \equiv$ St. uncertainty due to biological/sampling

4. $\mu_{b/s} = (0.0249 \cdot \overline{BrAC}_c) + 0.00173$

5. $U = k \cdot \mu_c$

Documents & Data
at back of packet

Result?
When we report a result simply as “a number,” it does not reflect the accuracy of the measurement and cannot be properly interpreted. Estimating and reporting measurement uncertainty with the number completes the picture and allows us to properly use the result to make reliable and defensible decisions.”

“All results for every forensic science method should indicate the uncertainty in the measurements that are made, and studies must be conducted that enable the estimation of those values.”

“For example, methods for measuring the level of blood alcohol in an individual or methods for measuring the heroin content of a sample can do so only within a confidence interval of possible values.”

“In particular, breath alcohol “results need to be reported, along with a confidence interval that has a high probability of containing the true blood-alcohol level (e.g., the mean plus or minus two standard deviations).”

Probability “true” BrAC is less than 0.08 g/210L?
**Error V. Uncertainty**

Measurement “error is an unknowable quantity in the realm of the *state of nature*…uncertainty (in measurement) is a quantifiable parameter in the realm of the *state of knowledge about nature.*”

The Issue of Below Threshold Interferents

*Threshold* = 0.01 \( \frac{g}{210L} \)

Error analysis → Bounded error

Uncertainty Analysis → Model knowledge with probability distributions

**Error Analysis**

*Bounded error* = 0.01 \( \frac{g}{210L} \)

Result?

**Uncertainty Analysis**

*Systematic effect* = \( \frac{2a}{2} \)

\[ \mu_i = \frac{a}{\sqrt{3}} \]

\[ \mu_c =? \]

Result?
“Scientific measurements provide a rigorous method for obtaining information about the physical world. No matter how sophisticated the measurement, though, there are inherent limitations on how narrowly it can characterize the value of a quantity of interest. If we fail to account for those limitations, then any inferences we make based on the results of measurement, whether or not they ultimately prove correct, are themselves inherently flawed. In the context of the criminal justice system, such flawed inferences can result in the innocent being deprived of their liberty and the guilty being exonerated. This not only undermines the integrity of the system and the verdicts it produces, but ultimately our belief that we are part of a fundamentally fair and just society.

Measurement uncertainty is important because it provides an explicit scientific statement of the limitations governing the rational inferences that can be made based upon the result of a particular scientific measurement. If we ignore this statement, then we are ignoring that which makes the measurement scientifically rigorous, thereby defeating the purpose for our reliance on it in the first place. Although the truth of a thing can never be known with absolute certainty, by adhering to the basic principles of science when using measurement as a tool in our search we can at least ensure that the strength of our belief in a proposition is supported by the available information.

In the end, we are requiring no more of forensic science than we are of the juror charged with the task of making a determination of guilt or innocence in the context of the necessarily imperfect and incomplete information provided at trial. No reasonable juror would ever claim to know the truth with absolute certainty, nor does our system require them to do so. Their charge is simply to consider the information provided, and only that provided, and determine whether their degree of belief concerning the question of guilt is strong enough to establish the proposition beyond a reasonable doubt. As in the measurement context, though, the integrity of the determination is only as good as the information it is based on. By supplying jurors with a measurement’s uncertainty, we empower them to make rational inferences and determinations. Failing to do so calls into question everything our society, our constitution and simple fairness asks them to do. And for that we all suffer.

Only when we recognize with the same fearless honesty of a child, that a measurement result unaccompanied by its uncertainty is no less naked than the Emperor who wore no clothes, can we begin to have confidence in verdicts based on the results of forensic measurements and in the basic integrity of justice rendered thereupon.”

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Introduction to Measurement Uncertainty
Tools & Concepts

Basic Measurement Concepts

Measurement: A set of empirical operations carried out to determine the quantity values that can reasonably be attributed to a quantity of interest. The objective of a measurement is to determine the value of the particular quantity being measured.

Measurand: The quantity intended to be measured.

Direct Measurement: A measurement that senses the quantity of interest itself and maps it to a quantity value without the necessity of intermediate determinations.

Indirect Measurement: The determination of a quantity of interest through its relationship to other directly measured quantities.

Accuracy: The degree of agreement of a measured value with the “true” value of the quantity of interest. The degree of agreement expected from a measurement method/instrument is typically determined by comparing the mean of a set of measurements of a reference standard to the accepted value of the reference standard. Whether a measurement or instrument/method is deemed accurate is not an absolute judgment. Rather, accuracy is judged with respect to the use to be made of the data. What might be deemed accurate in one set of circumstances may not be accurate in another.

Precision: Precision is concerned with the variability or scatter of the individual results of replicate measurements. Measurements that are tightly grouped are considered precise while those with greater scatter are less so. As was the case with accuracy, precision is judged with respect to the use to be made of the data. What may be considered precise for one purpose may not be precise for another.
Measurement Interpretation – I: If a measurement value is to be interpretable, we must have an understanding of how accurate and how precise the measurement is. Absent such information, a measured value is simply a number, the meaning of which we know little about. Ideally, important measurements would be both accurate and precise. That is, not only would such measurements yield mean values in close agreement with a “true” value, but individual values having a high degree of agreement with each other.

An objective characterization of accuracy and precision are necessary in order to determine the value of the particular quantity being measured. Such objective characterization can be supplied by statistics.

Basic Statistical Concepts

Population: The entire set or universe of objects sharing specific traits defining a class of objects.

Sample: A subset of objects selected from the population.

Distribution: The set of possible values of a random variable related through their frequency of occurrence or belief based relative likelihood.

Parameter: A characteristic of a population’s distribution.

Statistic: A characteristic of a sample’s distribution.

Descriptive Statistics: Utilizes data to describe the properties of a sample, not to make predictions based upon it.

Inferential Statistics: Utilizes data to draw inference or make predictions. A typical example is the use of sample data to generate a sample statistic from which an inference concerning a population parameter may be made.

Probability – Frequentist Interpretation: Probability is interpreted as relative frequency of occurrence over all sample data sets. As such, probabilities are objectively determined as a function of sampling data. Population parameters have unique, fixed true values that are unknown. The randomness lies in the sampling process, not the parameter. Since population parameters are nonrandom, probability statements cannot be made about their values. Nor can probability statements be made about a characteristic of a unique event. The parameter or characteristic either is or is not a particular value. The level of confidence associated with an inference refers to the confidence in the sampling/inferential process, not the actual quantity of interest. It tells us how often, over repeated samplings, our inference will happen to correspond to the true value.
Probability – Bayesian Interpretation: Probability is interpreted as an information-based “degree of belief” that an event will occur. Bayesian inference employs sampling data and any other information deemed relevant in the decision making process so that probability (degree of belief) may be based upon both objective and subjective components. In this framework, the parameters themselves are considered random so that probability statements can be made directly about their values. The same holds for a characteristic of a unique event. Thus, probability statements made concerning the value of a parameter or characteristic are about the actual quantity of interest. It tells us the probability that this particular inference is “true”.

**Measurement Error**

Measurement Error: Traditionally, the quality of a measurement result was addressed through error analysis. This approach considered each measurand as having a unique true value. “The objective of measurement in the Error Approach is to determine an estimate of the true value that is as close as possible to that single true value. The deviation from the true value is composed of random and systematic errors.”

Systematic Error: The tendency of a set of measurements to consistently (on average) underestimate or overestimate the “true” value of the measurand by a given value or percentage. Most measurements have some amount of systematic error associated with them. Systematic error may be related to measuring methods, instruments or even empirically based calculations. It is a primary component of accuracy as it has a direct and regular impact on the degree of agreement of a measured value with the “true” value of the quantity of interest. Accordingly, “if a systematic error has not been accounted for, all [measured] values could be misleading.” Fortunately, once identified systematic error can be corrected for. “It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.”

Random Error: The unpredictable/random fluctuation in measurement results under fixed conditions. Random error is associated with precision. Unlike systematic error, random error cannot be corrected for. It is an inherent aspect of all measurement results. Although random error cannot be completely eliminated, it can be minimized by making a large number of measurements.
**Arithmetic mean:** This is a simple average of measurement values. It is determined by adding all measured values together and then dividing the sum by the number of values included in the sum. It is typically used when all measured values are considered to be equally reliable.

\[
\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i
\]

**Bias:** Quantitative measure of systematic error. Bias is typically treated as either having a constant magnitude across a range of measured values or being proportional to the measured value obtained. When proportional, the bias is commonly reported as a percent bias. For chemical measurements, it is not uncommon for the bias to be proportional to measured values. “Whenever the true value of the measured quantity is needed...bias can be a serious problem.” Fortunately, once bias has been determined, systematic error can be easily accounted for. The bias of a method or instrument is ordinarily determined by comparing the mean of a set of measurements of a reference standard to its accepted value.

\[
b_c = \bar{y} - Y_{ref}
\]

\[
b\% = \frac{\bar{y} - Y_{ref}}{Y_{ref}}
\]

**Standard Deviation:** Quantitative characterization of the variability/dispersion of individually measured values about their mean. The standard deviation is the root mean square deviation of measured values from their mean. Precision/random error is typically expressed in terms of a standard deviation. The determination of the standard deviation varies slightly depending on the source of our data. If the standard deviation has been determined from a population, we use what is commonly referred to as a population standard deviation. On the other hand, when our data comes from a sample, we use what is commonly referred to as a sample standard deviation. Throughout the remainder of this section the distinction will not be noted unless necessary but it is assumed that whenever employed, the correct standard deviation is utilized.

\[
\sigma_p = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} (\bar{y} - y_i)^2}
\]

\[
\sigma_s = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (\bar{y} - y_i)^2}
\]
Coefficient of Variation: The standard deviation expressed as a proportion relative to the mean of a set of measurements. The coefficient of variation can be useful when combining standard deviations or comparing the variability of separate measurements.

\[ cv = \frac{\sigma}{\bar{y}} \]

Bias Adjusted Mean/Best Estimate of True Value: The mean adjusted for bias. The bias adjusted mean is often considered the best estimate of the “true” value of the measurand. Whenever reporting the mean of a set of measurement, it should be corrected for bias. The correction applied depends upon whether the bias is constant or proportional.

\[ \bar{y}_b = \bar{y} - b_c \]
\[ \bar{y}_b = \frac{\bar{y}}{1 + b\%} \]

Confidence interval: A range of values symmetric about the bias adjusted mean constructed using a multiple of the standard deviation of the set of measurements and expected to cover the true value with a given level of confidence (likelihood).

\[ C.I. = \bar{y}_b \pm k\sigma_y \]

The likelihood that the interval will overlap the true value is determined by the multiplier of the standard deviation \( (k) \), known as a coverage factor, and the underlying distribution. If the underlying distribution is Gaussian (normal) the likelihood associated with \( k = 1,2 & 3 \), is given in the following figure.

One should be very careful with the interpretation of a confidence interval. The focus of the level of confidence is not the true value. That is, the level of confidence does not refer to the probability that the true value lies within the interval. It either does or does not. Rather, the subject of the level of confidence is the sampling procedure. It tells you that based upon the procedure utilized, you will be able to construct an interval that will overlap the true value a given percent of the time. In technical terms, “[t]he confidence reflects the proportion of
cases that the confidence interval would contain the true parameter value in a long series of repeated random samples under identical conditions. The confidence interval is based upon frequentist philosophy and the existence of a singular true value.

**Standard deviation (error) of the mean:** Quantitative characterization of the variability/dispersion of sample means. Due to the Central Limit Theorem, the following relationship holds regardless of the underlying population distribution as long as the sample size is large enough.

\[ \sigma_y = \frac{\sigma_y}{\sqrt{N}} \]

**Weighted mean:** The weighted mean is an alternative way to determine the best estimate of the true value of a measurand. When combining multiple values determined for a given measurand, a weighted mean attaches more weight to those values considered more reliable.

\[ \bar{y}_{wm} = \frac{\sum_{i=1}^{N} w_i y_i}{\sum_{i=1}^{N} w_i} \]

**Traditional weighted mean:** Frequently, the values to be combined are the arithmetic means from several sets of measurements. The traditional weighted mean relies upon the precision associated with each set of measurements to determine the weight to accord the mean associated with each set. The greater the precision associated with a given mean, the more confidence we have in the value, and the more weight it is accorded in combining the means to determine a best estimate of the true value. In this case the above expression becomes:

\[ \bar{y}_{wm} = \frac{\sum_{i=1}^{N} \frac{n_i}{\sigma_i^2} y_i}{\sum_{i=1}^{N} \frac{n_i}{\sigma_i^2}} \]

The weighted mean should be employed when the values to be combined are not equally reliable.

**Standard deviation of the Traditional Weighted Mean:**

\[ \sigma_{wm} = \frac{1}{\sqrt{\sum \frac{n_i}{\sigma_i^2}}} \]

**Measurement Interpretation – II:** If a measurement value is to be interpretable, we must have a quantitative determination of the systematic and random error associated with the measurement. Absent such information, a measured value is simply a number, the meaning of which we know little about. It has long been understood that no measurement result can be interpreted where only the value of the measurement itself is reported. Proper interpretation of a measured value requires knowledge and incorporation of the measurement’s systematic and random error into any reported values.

Unfortunately, as useful as traditional error analysis is, “[i]t is now widely recognized that, when all of the known or suspected components of error have been evaluated and the
appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the result of the measurement represents the value of the quantity being measured.  

Put simply, it is not possible to know the true value of a measurand or the error of a measurement result and hence how close a measurement result is to the true measurand value.

**Measurement Uncertainty**

**Measurement Uncertainty**: “[F]or a given measurand and a given result of measurement of it, there is not one value but an infinite number of values dispersed about the result that are consistent with all of the observations and data and one’s knowledge of the physical world, and that with varying degrees of credibility can be attributed to the measurand.” Accordingly, “[t]he objective of measurement in the Uncertainty Approach is not to determine a true value as closely as possible. Rather, it is assumed that the information from measurement only permits assignment of an interval of reasonable values to the measurand, based on the assumption that no mistakes have been made in performing the measurement.”

Contrary to the traditional approach, then, the measurand is not treated as having a unique “true” value. Instead, the measurand is deemed to consist of a set of “true” values. Measurement uncertainty is a quantitative statement characterizing the dispersion of values that can actually and reasonably be attributed “to a measurand based on the information available including systematic and random effects…and any other factors that may impact the measurement or test process or result.” Measurement uncertainty is based upon the Bayesian notion of probability as a measure of degree of belief.

**Standard Uncertainty**: The total uncertainty associated with any measurement result is typically the result of the combination of several smaller uncertainties associated with particular aspects of the measurement process. Each component of uncertainty that contributes to the uncertainty of a measurement result is known as a standard uncertainty. Each standard uncertainty is expressed and treated as, and may in fact be, a standard deviation.

\[ \mu \equiv \sigma \]

**Relative Standard Uncertainty**: The standard uncertainty expressed as a proportion relative to the mean of a set of measurements. It can be useful when combining standard uncertainties or comparing the uncertainty of separate measurements.

\[ \mu_r = \frac{\mu_y}{\bar{y}_b} \]

**Type A Uncertainty**: Component of uncertainty that has been determined by the statistical analysis of measured values. Determination is based on frequency distributions and any statistically valid method for data analysis. An example is the standard deviation determined from a set of measurements.

**Type B Uncertainty**: Component of uncertainty that has been determined by means other than the statistical analysis of measured values. Determination assumes *a priori* distributions based on relevant information and scientific judgment. Examples include information provided by instrument manufacturer, metrological certifications and reference publications.
**Combined Uncertainty:** The combination of all the standard uncertainties associated with a measurement. The individual standard uncertainties are combined in the same manner as standard deviations. Assuming the standard uncertainties are random and independent, the combined uncertainty is the root sum square of the standard uncertainties. The combined uncertainty is expressed and treated as, and may in fact be, a standard deviation.

\[
\mu_c = \sqrt{\sum_{i=1}^{n} \mu_i^2}
\]

When determining the combined uncertainty of a measurement it is critical to include all significant components of uncertainty. Failure to do so will cause an underestimate of the uncertainty misleading others to believe that the result is more precise than it actually is.

**Expanded Uncertainty:** Obtained by multiplying the combined uncertainty by a coverage factor.

\[
U = k\mu_c
\]

A coverage factor is chosen such that when the expanded uncertainty is expressed as part of a complete measurement result it conveys a range of values that can actually and reasonably be attributed to a measurand with a given level of confidence. The level of confidence associated with a given coverage factor is determined by the measurement’s underlying distribution. If the underlying distribution is Gaussian (normal) the level of confidence associated with \( k = 1.64, 1.96 & 2.576 \) is given in the following table.

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<td>1.96</td>
<td>95%</td>
</tr>
<tr>
<td>2.576</td>
<td>99%</td>
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</table>

**Measurement Result:** “In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement, that is, the measurand, and thus the result is complete only when accompanied by a quantitative statement of its uncertainty.”\(^{52}\) Moreover, “[i]t is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.”\(^{53}\) Accordingly, a complete measurement result consists of the best estimate of the true value of the measurand, typically the bias adjusted mean, accompanied by the expanded uncertainty and its associated level of confidence.

\[
Y = \bar{y}_b \pm U (99\%)
\]

This is interpreted to mean that the best estimate of the value attributable to the measurand \( Y \) is \( \bar{y}_b \), and that \( \bar{y}_b - U \) to \( \bar{y}_b + U \) is the range of values that could actually be attributed to \( Y \) with a 99% level of confidence.
**Coverage Interval**: An “interval containing the set of true quantity values of a measurand with a stated probability, based on the information available.” Ordinarily the coverage interval is derived from the expanded uncertainty and is symmetric about the mean so that it can be expressed as:

\[
\mathcal{C} = \bar{y}_b \pm U (99\%)
\]

Note that the coverage interval is identical to the measurement result. Unlike the confidence interval, the coverage interval is based upon Bayesian philosophy so that it refers directly to the quantity of interest, the “true” value of the measurand. In this context, the level of confidence is the probability, understood as a degree of belief, “that the set of true quantity values of a measurand is contained within a specified coverage interval.” It should also be noted that the coverage interval need not be symmetric about the mean.

**Measurement Interpretation – III**: For even the most carefully performed measurement, a unique “true” value for a measurand can never be determined. All that can ever be given is a set of values, all of which may actually and reasonably be assigned as “true” values. If a measurement value is to be interpretable, it must be corrected for bias and accompanied by a *quantitative estimate of its uncertainty*. Absent such information, a measured value is simply a number, the meaning of which we know little about.

“Knowledge of the uncertainty associated with measurement results is essential to the interpretation of the results. Without quantitative assessments of uncertainty, it is impossible to decide whether observed differences between results reflect more than experimental variability, whether test items comply with specifications, or whether laws based on limits have been broken. Without information on uncertainty, there is a risk of misinterpretation of results. Incorrect decisions taken on such a basis may result in unnecessary expenditure in industry, incorrect prosecution in law, or adverse health or social consequences.”

**Functional Relationships, Measurement Functions and Propagation of Uncertainty**

**Algorithmic Determinations**: When the quantity of interest cannot be measured directly, we must rely upon mathematical relationships between the quantity of interest and other measured and/or “given” values to calculate the quantity of interest. Each measured value and many “given” values have uncertainty associated with them. These uncertainties propagate through the calculation and are imparted to the value determined for the quantity of interest.

**Measurement Function – General Form**: A functional relationship between the quantity of interest and the input quantities (measured and/or “given” values) needed to calculate it.

\[ Y = f(X, W \cdots Z) \]

**Best Estimate of True Value – General Form**: The best estimate of a quantity value based on a measurement function is given by plugging in the best estimate for each of the input quantities

\[ Y_b = f(x_b, w_b \cdots z_b) \]
Propagation of Uncertainty – General Form: For a quantity value based upon a general measurement function:

\[
\mu_y = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \cdot \mu_{x_i} \right)^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot \mu_{x_i x_j}}
\]

\[
\mu_y = \sqrt{\left( \frac{\partial f(X, Z)}{\partial X} \Bigg|_{X=X_b, Z=Z_b} \cdot \mu_x \right)^2 + \left( \frac{\partial f(X, Z)}{\partial Z} \Bigg|_{X=X_b, Z=Z_b} \cdot \mu_z \right)^2 + 2 \cdot \frac{\partial f(X, Z)}{\partial X} \Bigg|_{X=X_b, Z=Z_b} \cdot \frac{\partial f(X, Z)}{\partial Z} \Bigg|_{X=X_b, Z=Z_b} \cdot \mu_{xz}}
\]

\[
\mu_y \leq \left| \left. \frac{\partial f(X, \ldots, Z)}{\partial X} \right|_{X=X_b, \ldots, Z_b} \cdot \mu_x + \ldots + \left| \left. \frac{\partial f(X, \ldots, Z)}{\partial Z} \right|_{X=X_b, \ldots, Z_b} \cdot \mu_z \right|
\]

Independent Input Quantities

\[
\mu_y = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \cdot \mu_{x_i} \right)^2}
\]

\[
\mu_y = \sqrt{\left( \frac{\partial f(X, \ldots, Z)}{\partial X} \Bigg|_{X=X_b, \ldots, Z_b} \cdot \mu_x \right)^2 + \ldots + \left( \frac{\partial f(X, \ldots, Z)}{\partial Z} \Bigg|_{X=X_b, \ldots, Z_b} \cdot \mu_z \right)^2}
\]

Covariance: A measure of the association between two random variables. If two input quantities are independent then the covariance will be zero. When two input quantities are not independent this term appears in the propagation of uncertainty calculation to account for the dependence.

\[
\mu_{xy} = \frac{1}{N} \cdot \sum_{i=1}^{N} (\bar{x}_i - x_i)(\bar{y}_i - y_i)
\]

Measurement Function – Measured quantity multiplied by a constant:

\[
Y = a \cdot X
\]

\[
Y_b = a \cdot x_b
\]

\[
\mu_y = a \cdot \mu_x
\]
Measurement Function – Variable raised to a constant power:

\[ Y = X^n \]

\[ Y_b = x_b^n \]

\[
\mu_{ry} = \frac{\mu_y}{|Y_b|} = |n| \frac{\mu_x}{|x_b|}
\]

Measurement Function – Sums and differences:

\[ Y = X - W + \cdots + Z \]

\[ Y_b = x_b - w_b + \cdots + z_b \]

Independent

\[
\mu_y = \sqrt{\mu_x^2 + \mu_w^2 + \cdots + \mu_z^2}
\]

All Circumstances

\[
\mu_y \leq \mu_x + \mu_w + \cdots + \mu_z
\]

Measurement Function – Products and quotients:

\[ Y = \frac{X \times \cdots \times W}{Z \times \cdots \times Q} \]

\[ Y_b = \frac{x_b \times \cdots \times w_b}{z_b \times \cdots \times q_b} \]

Independent

\[
\mu_{ry} = \frac{\mu_y}{|Y_b|} = \sqrt{\left(\frac{\mu_x}{x_b}\right)^2 + \left(\frac{\mu_w}{w_b}\right)^2 + \cdots + \left(\frac{\mu_z}{z_b}\right)^2 + \left(\frac{\mu_q}{q_b}\right)^2}
\]

All Circumstances

\[
\mu_{ry} \leq \frac{\mu_y}{|Y_b|} \leq \frac{\mu_x}{|x_b|} + \frac{\mu_w}{|w_b|} + \cdots + \frac{\mu_z}{|z_b|} + \frac{\mu_q}{|q_b|}
\]

**Examples: BAC Results & Calculations**

Breath Testing: Like any other measurement, forensic breath alcohol concentration tests have both bias and uncertainty associated with them. Both need to be determined and incorporated into a complete test result.

Best estimate for “true” BrAC (Bias adjusted mean): \((\overline{BrAC}_b)\)

Best estimate for “true” BrAC determined by computing the bias adjusted mean.
Machine bias: \((b_M)\)
Determined during calibration and which will be deemed proportional to the concentration measured.

Interferent bias: \((b_I)\)

Most breath test instruments are designed to detect the presence of interferents on an individual’s breath. However, some are programmed such that they will only do so if the interferent exceeds a particular level. There are several ways one might try to determine the average impact/bias such interferent will have on a breath test below the level of detection but which will nonetheless contribute to the reported value. One could consult the literature to determine if there are published values. Another way is to postulate an underlying distribution based upon all the known information and determine the mean (expected) contribution due to bias based on the distribution. The bias due to this source will be a constant offset.

Best estimate for “true” BrAC (Bias adjusted mean):

\[
\overline{BrAC}_b = \frac{BrAC}{1 + b_M} - b_I
\]

Combined uncertainty for BAC based on measurement:\(^57\) \((\mu_{BrAC})\)
There are several sources of uncertainty that may be associated with a breath test. The ones considered here comprise only a subset and may or may not be relevant to your test. For ease of illustration they are treated as being independent.

Reference material: \((\mu_r)\)
The standard uncertainty associated with the reference material utilized to calibrate machine.

Machine precision: \((\mu_o)\)
The precision of the breath test machine determined at the time of it’s calibration and expressed as a standard uncertainty.

Bias: \((\mu_b)\)
The standard uncertainty associated with the value determined for the bias.

Sampling: \((\mu_s)\)
The standard uncertainty due to circumstances arising during the collection of breath samples.

Combined uncertainty for BAC based on measurement:

\[
\mu_{BrAC} = BrAC_b \cdot \sqrt{\left(\frac{\mu_r}{r_b}\right)^2 + \left(\frac{\mu_o}{\sigma_b}\right)^2 + \left(\frac{\mu_b}{b_b}\right)^2 + \left(\frac{\mu_s}{s_b}\right)^2}
\]
Complete Result:

\[
BrAC = \overline{BrAC} \pm k\mu_{\epsilon_{BrAC}}
\]

Breath as an indirect measure of blood: When the breath alcohol concentration is being utilized as an indirect measure of blood alcohol concentration, the breath result must be converted into one for blood. This involves a conversion factor \( M \) between breath and blood alcohol concentration which the literature illustrates has a great deal of uncertainty associated with it. This uncertainty must also be factored into a reported result.

Functional Relationship:

\[
BAC = M \cdot BrAC
\]

Combined Uncertainty:

\[
\mu_{\epsilon_{BAC}} = \sqrt{\left(\frac{\partial BAC(M, BrAC)}{\partial M} \cdot \mu_M\right)^2 + \left(\frac{\partial BAC(M, BrAC)}{\partial BrAC} \cdot \mu_{\epsilon_{BrAC}}\right)^2 + 2 \cdot \frac{\partial BAC(M, BrAC)}{\partial M} \cdot \frac{\partial BAC(M, BrAC)}{\partial BrAC} \cdot \mu_{\epsilon_{M,BrAC}}}
\]

\[
= \sqrt{(BrAC_b \cdot \mu_M)^2 + (M_b \cdot \mu_{\epsilon_{BrAC}})^2 + 2 \cdot BrAC_b \cdot M_b \cdot \mu_{\epsilon_{M,BrAC}}}
\]

Complete Result:

\[
BAC_b = M_b \cdot BrAC_b \pm k\mu_{\epsilon_{BAC}}
\]

Widmark’s Formula: For the determination of blood alcohol content based on the number of drinks consumed.

Functional Relationship 1: Assuming post-absorptive.

\[
C_t = \frac{NdZ}{W\tau} - \beta t
\]

The Variables:

- \( C_t \equiv \text{BAC at time } t \)
- \( W \equiv \text{Body weight} \)
- \( N \equiv \text{Number of drinks} \)
- \( \tau \equiv \text{Volume of distribution} \)
- \( d \equiv \text{Density of alcohol} \)
- \( \beta \equiv \text{Alcohol elimination rate} \)
- \( Z \equiv \text{Ethanol per drink} \)
- \( t \equiv \text{Time since drinking began} \)
Combined Uncertainty:

\[ \mu_{C_t} = \sqrt{\left[ \sum \left( \frac{\partial C_t}{\partial x_i} \cdot \mu_{x_i} \right)^2 \right] + 2 \cdot \frac{\partial C_t}{\partial \tau} \cdot \frac{\partial C_t}{\partial \beta} \cdot \mu_{\tau,\beta}} \]

\[ = \sqrt{\left( \frac{dZ}{W^2} \cdot \mu_d \right)^2 + \left( \frac{NZ}{W^2} \cdot \mu_p \right)^2 + \left( \frac{NdZ}{W^2} \cdot \mu_z \right)^2 + \left( \frac{NdZ}{W^2} \cdot \mu_z \right)^2 + \left( \frac{\beta}{\tau} \right)^2 + \left( \frac{\beta \cdot \mu_d}{\tau} \right)^2 + 2 \cdot \left( \frac{NdZ}{W^2} \right) (\tau) \cdot \mu_{\tau,\beta} } \]

Complete Result:

\[ C_{tb} = \frac{N_b d_b Z_b}{W_b \tau_b} - \beta_b t_b \pm k \mu_{C_t} \]

Functional Relationship 2: Accounting for rate of absorption.

\[ C_t = \frac{N d Z (1 - e^{-\gamma t})}{W \tau} - \beta t \]

The Variables:

- \( C_t \equiv \) BAC at time \( t \)
- \( W \equiv \) Body weight
- \( N \equiv \) Number of drinks
- \( \tau \equiv \) Volume of distribution
- \( d \equiv \) Density of alcohol
- \( Z \equiv \) Ethanol per drink
- \( \gamma \equiv \) Alcohol absorption rate
- \( \beta \equiv \) Alcohol elimination rate
- \( t \equiv \) Time since drinking began

Combined Uncertainty:

\[ \mu_{C_t} = \sqrt{\left[ \sum \left( \frac{\partial C_t}{\partial x_i} \cdot \mu_{x_i} \right)^2 \right] + 2 \cdot \frac{\partial C_t}{\partial \tau} \cdot \frac{\partial C_t}{\partial \beta} \cdot \mu_{\tau,\beta}} \]

\[ = \sqrt{\left( \frac{dZ(1 - e^{-\gamma t})}{W^2} \cdot \mu_d \right) + \left( \frac{N(1 - e^{-\gamma t})}{W} \cdot \mu_p \right) + \left( \frac{Nd(1 - e^{-\gamma t})}{W^2} \cdot \mu_z \right) + \left( \frac{Nd(1 - e^{-\gamma t})}{W^2} \right) \cdot \mu_{\tau,\beta} + \left( \frac{\beta}{\tau} \right)^2 + \left( \frac{\beta \cdot \mu_d}{\tau} \right)^2 + 2 \cdot \left( \frac{Nd(1 - e^{-\gamma t})}{W^2} \right) (\tau) \cdot \mu_{\tau,\beta} } \]

Complete Result:

\[ C_{tb} = \frac{N_b d_b Z_b(1 - e^{-\gamma b t_b})}{W_b \tau_b} - \beta_b t_b \pm k \mu_{C_t} \]
EXERCISE

BREATH ALCOHOL UNCERTAINTY: WASHINGTON STATE

The “True” BrAC

<table>
<thead>
<tr>
<th>--- BREATH ANALYSIS ---</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLANK TEST</td>
</tr>
<tr>
<td>INTERNAL STANDARD</td>
</tr>
<tr>
<td>SUBJECT SAMPLE</td>
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<tr>
<td>BLANK TEST</td>
</tr>
<tr>
<td>EXTERNAL STANDARD</td>
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<tr>
<td>BLANK TEST</td>
</tr>
<tr>
<td>SUBJECT SAMPLE</td>
</tr>
<tr>
<td>BLANK TEST</td>
</tr>
</tbody>
</table>

Complete BrAC Result = Best Estimate ± Expanded Uncertainty

\[
BrAC = BrAC_c \pm U (99\%)
\]

Mean Measured Value (\(BrAC\))

\[
\bar{BrAC} = \frac{brac_1 + brac_2}{2}
\]

\[
= \frac{0.082 + 0.088}{2}
\]

\[
= 0.0850
\]
Bias Corrected Mean/Best Estimate ($\overline{BrAC_c}$)

$$\overline{BrAC_c} = \frac{BrAC}{1 + (b \cdot 0.01)}$$

**Systematic Effect/Bias ($b$)**

```
<table>
<thead>
<tr>
<th>Reference Value</th>
<th>CERTIFICATION RESULTS (g/210L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>QAP Batch #</td>
<td>10020</td>
</tr>
<tr>
<td>Simulator #</td>
<td>G7009</td>
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<tr>
<td>Sim Thermometer #</td>
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<tr>
<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>0.040</td>
</tr>
<tr>
<td>7</td>
<td>0.040</td>
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<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
<td>0.040</td>
</tr>
<tr>
<td>10</td>
<td>0.059</td>
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<tr>
<td>Mean</td>
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<tr>
<td>SD</td>
<td>0.0005</td>
</tr>
<tr>
<td>Bias %</td>
<td>1.27</td>
</tr>
<tr>
<td>CV %</td>
<td>1.25</td>
</tr>
</tbody>
</table>
```

$$\overline{BrAC_c} = \frac{BrAC}{1 + (b \cdot 0.01)}$$

$$\overline{BrAC_c} = \frac{0.0850}{1 + (3.02 \cdot 0.01)}$$

$$= 0.0825$$
Combined Uncertainty for BrAC ($\mu_c$)

$$
\mu_c = \overline{BrAC_c} \sqrt{\left(\frac{QAP_{SD}/\sqrt{10}}{QAP_{mean}}\right)^2 + \left(\frac{\mu_{Qsol}}{Qsol_{ref}}\right)^2 + \left(\frac{\mu_{b/s}/\sqrt{2}}{BrAC_c}\right)^2}
$$

where

$\overline{BrAC_c} \equiv$ bias corrected mean

$QAP_{SD} \equiv$ QAP standard deviation

$QAP_{mean} \equiv$ QAP mean

$\mu_{Qsol} \equiv$ Standard uncertainty of QAP solution

$Qsol_{ref} \equiv$ Vapor concentration of QAP solution

$\mu_{b/s} \equiv$ Standard uncertainty due to biological/sampling

**QAP$_{SD}$ & QAP$_{mean}$**

![Image of Datamaster Calibration Certificate]

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Introduction to Measurement Uncertainty
\( Q_{sol\, ref} \) & \( \mu_{Q_{sol}} \)

**Washington State Patrol - Toxicology Laboratory Division**

**QAP Test Report Calculation Record**

<table>
<thead>
<tr>
<th>Analyst</th>
<th>NN</th>
<th>BP</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>HS/3</td>
<td>HS/3</td>
<td>HS/3</td>
</tr>
<tr>
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<td>0.097</td>
<td>0.098</td>
</tr>
<tr>
<td>2</td>
<td>0.098</td>
<td>0.097</td>
<td>0.098</td>
</tr>
<tr>
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<td>0.098</td>
<td>0.097</td>
<td>0.098</td>
</tr>
<tr>
<td>4</td>
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<td>0.097</td>
<td>0.098</td>
</tr>
<tr>
<td>5</td>
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<td>0.097</td>
<td>0.098</td>
</tr>
<tr>
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<td>0.097</td>
<td>0.098</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CV(^2)(_{CDA})</th>
<th>CV(^2)(_{QAP, Solution})</th>
<th>CV(^2)(_{CV, Control})</th>
<th>CV(^2)(_{Bias})</th>
<th>CV(^2)(_{Part, Cost})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000084100</td>
<td>0.0000021906</td>
<td>0.0000114134</td>
<td>0.0001333333</td>
<td>0.0001016326</td>
</tr>
</tbody>
</table>

Average Solution Concentration: 0.0978 g/100mL
Standard Deviation: 0.00056 g/100mL
Precision CV (%): 0.57
Equivalent Vapor Concentration: 0.0795 L-210L
Combined Standard Uncertainty (\(\mu\)): 0.0013 \(\mu\)L-210L

\[ (\mu_{b/s}) \]

\[ \mu_{b/s} = (0.0249 \cdot BrAC_c) + 0.00173 \]

\[ \mu_{b/s} = (0.0249 \cdot 0.0825) + 0.00173 \]

\[ = 0.00378 \]

**Combined Uncertainty for BrAC (\(\mu_c\))**

\[ \mu_c = BrAC_c \sqrt{\left(\frac{QAP_{SD}/\sqrt{10}}{QAP_{mean}}\right)^2 + \left(\frac{\mu_{Q_{sol}}}{Q_{sol\, ref}}\right)^2 + \left(\frac{\mu_{b/s}/\sqrt{2}}{BrAC_c}\right)^2} \]

\[ \mu_c = 0.0825 \sqrt{\left(\frac{0.0007/\sqrt{10}}{0.0819}\right)^2 + \left(\frac{0.0013}{0.0795}\right)^2 + \left(\frac{0.00378/\sqrt{2}}{0.0825}\right)^2} \]

\[ = 0.0825 \sqrt{0.00007 + 0.000267 + 0.00105} \]

\[ = 0.00300 \]
Expanded Uncertainty ($U$)

\[ U = k \cdot \mu_c \]

\[ k = 2.576 \rightarrow 99\% \]

\[ U = 2.576 \cdot 0.00300 \]

\[ = 0.00773 \]

Complete BrAC Result (BrAC) = Best Estimate ± Expanded Uncertainty

\[ BrAC = \frac{BrAC_c + U}{99\%} \]

\[ BrAC = 0.0825 \pm 0.00773 \cdot \frac{g}{210L} (99\%) \]

Coverage Interval

\[ 0.0748 \cdot \frac{g}{210L} \leftarrow BrAC_{99\%} \rightarrow 0.0902 \cdot \frac{g}{210L} \]
Probability “True” BrAC is less than 0.08 g/210L

Step 1: Coverage interval.

\[ 0.0748 \leftrightarrow 0.0902 \]

Step 2: Bias corrected mean

\[ \overline{BrAC}_c = \frac{(b_1 + b_2)}{2} = \frac{(0.0748 + 0.0902)}{2} = 0.0825 \]

Step 3: Expanded uncertainty

\[ U = \overline{BrAC}_c - b_1 = 0.0825 - 0.0748 = 0.0077 \]

Step 4: Combined uncertainty

\[ \mu_c = \frac{U}{2.576} = \frac{0.0077}{2.576} = 0.00299 \]

Step 5: Z-factor

\[ z_{\overline{BrAC}_c-.08} = \frac{(\overline{BrAC}_c-.08)}{\mu_c} = \frac{(0.0825-.08)}{0.00299} = 83.6 \]

Step 6: Probability associated with the z-factor

\[ P_{\overline{BrAC} <.08} \approx 20\% \] [\textit{Z.836\rightarrow tail Table}]

| \( |z| \) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|------|------|------|------|------|------|------|------|------|
| 0.0 | 0.0000 | 0.0400 | 0.0900 | 0.1400 | 0.1900 | 0.2400 | 0.2900 | 0.3400 | 0.3900 |
| 0.1 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 |
| 0.2 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 | 0.4000 |
| 0.3 | 0.3000 | 0.3000 | 0.3000 | 0.3000 | 0.3000 | 0.3000 | 0.3000 | 0.3000 | 0.3000 |
| 0.4 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 |
| 0.5 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 |
| 0.6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.3 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.4 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.3 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.4 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

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Introduction to Measurement Uncertainty
Uncertainty Due to Instrumental Precision

v.

Uncertainty Due to Systematic Correction

Standard Uncertainty – Instrumental Precision \((QAP_{SD})\)

\[ \mu_I = QAP_{SD} \]

Standard Uncertainty – Systematic Correction

\[ \mu_b = \frac{QAP_{SD}}{\sqrt{10}} \]
Current Algorithm Combined Uncertainty for BrAC ($\mu_c$)

$$\mu_c = \frac{BrAC_c}{\sqrt{\left(\frac{QAP_{SD}}{QAP_{mean}}\right)^2 + \left(\frac{\mu_{Qsol}}{Q_{sol_{ref}}\sqrt{10}}\right)^2 + \left(\frac{\mu_{b/s}/\sqrt{2}}{BrAC_c}\right)^2}}$$

Corrected Algorithm Combined Uncertainty for BrAC ($\mu_c$)

$$\mu_c = \frac{BrAC_c}{\sqrt{\left(\frac{QAP_{SD}}{QAP_{mean}}\right)^2 + \left(\frac{QAP_{SD}/\sqrt{10}}{QAP_{mean}}\right)^2 + \left(\frac{\mu_{Qsol}}{Q_{sol_{ref}}\sqrt{10}}\right)^2 + \left(\frac{\mu_{b/s}/\sqrt{2}}{BrAC_c}\right)^2}}$$

Corrected Combined Uncertainty for BrAC ($\mu_c$)

$$\mu_c = 0.0825 \sqrt{\left(\frac{0.0007}{0.0819}\right)^2 + \left(\frac{0.0007/\sqrt{10}}{0.0819}\right)^2 + \left(\frac{0.0013}{0.0795}\right)^2 + \left(\frac{0.00378/\sqrt{2}}{0.0825}\right)^2}$$

$$= 0.0825 \sqrt{0.00073 + 0.00007 + 0.000267 + 0.00105}$$

$$= 0.00308$$

Corrected Expanded Uncertainty for BrAC ($U$)

$$U = 2.576 \cdot 0.00308$$

$$= 0.00793$$

Corrected Result

$$BrAC = 0.0825 \pm 0.00793 \frac{g}{210L} (99\%)$$

Corrected Coverage Interval

$$0.0746 \frac{g}{210L} \leftarrow BrAC_{99\%} \rightarrow 0.0904 \frac{g}{210L}$$
**Uncertainty, Type B Evaluation and Error**

**Interferent Threshold – Systematic Effect**

\[ \text{Threshold} = 0.01 \text{ g/210L} \]

**Asymmetric Distribution and Propagation of Distributions**

*The Monte Carlo Technique*

**Symmetric Approximation and GUM Type B Evaluation**

*The Uniform Distribution*

**Correction Due to Systematic Effect of Interferent Threshold**

\[ c_{sys:uni} = \frac{(2a)}{2} = \frac{.01}{2} \text{ g/210L} = .005 \text{ g/210L} \]
Corrected Best Estimate

\[ BrAC_b = BrAC_c - C_{sys:uni} \]
\[ = 0.0825 - 0.005 \]
\[ = 0.0775 \]

Standard Uncertainty Due to Correction

\[ \mu_i = \frac{a}{\sqrt{3}} \]
\[ = \frac{0.005}{\sqrt{3}} \]
\[ = 0.00289 \]

Corrected Combined Uncertainty for BrAC (\( \mu_c \))

\[ \mu_c = BrAC_b \sqrt{ \left( \frac{QAP_{SD}}{QAP_{mean}} \right)^2 + \left( \frac{QAP_{SD}/\sqrt{10}}{QAP_{mean}} \right)^2 + \left( \frac{\mu_{Qsol}}{Q_{sol_{ref}}} \right)^2 + \left( \frac{\mu_{b/s}/\sqrt{2}}{BrAC_b} \right)^2 + \left( \frac{\mu_i/\sqrt{2}}{BrAC_b} \right)^2 } \]
\[ \mu_c = 0.0775 \sqrt{ \left( \frac{0.0007}{0.0819} \right)^2 + \left( \frac{0.0007/\sqrt{10}}{0.0819} \right)^2 + \left( \frac{0.0013}{0.0795} \right)^2 + \left( \frac{0.00378/\sqrt{2}}{0.0775} \right)^2 + \left( \frac{0.00289/\sqrt{2}}{0.0775} \right)^2 } \]
\[ = 0.0775 \sqrt{0.000073 + 0.000007 + 0.000267 + 0.00105 + 0.000695} \]
\[ = 0.00355 \]

Corrected Expanded Uncertainty for BrAC (\( U \))

\[ U = 2.576 \cdot 0.00355 \]
\[ = 0.00915 \]
**Corrected Result**

\[ \text{BrAC} = 0.0775 \pm 0.00915 \frac{g}{210L} (~99\%) \]

**Corrected Coverage Interval**

\[ 0.0684 \frac{g}{210L} \leftarrow \text{BrAC}_{99\%} \rightarrow 0.0867 \frac{g}{210L} \]
**Probability “True” BrAC is less than 0.08 g/210L**

Step 1: Coverage interval.

\[ 0.0684 \leftrightarrow 0.0867 \]

Step 2: Bias corrected mean

\[ \overline{BrAC}_c = \frac{(b_l + b_u)}{2} = \frac{0.0684 + 0.0867}{2} = 0.0775 \]

Step 3: Expanded uncertainty

\[ U = \overline{BrAC}_c - b_l = 0.0775 - 0.0684 = 0.0091 \]

Step 4: Combined uncertainty

\[ \mu_c = \frac{U}{2.576} = \frac{0.0091}{2.576} = 0.00353 \]

Step 5: Z-factor

\[ z_{BrAC_{-0.08}} = \frac{(\overline{BrAC}_{-0.08})}{\mu_c} = \frac{(0.0775 - 0.08)}{0.00353} = -0.708 \]

Step 6: Probability associated with the z-factor

\[ P_{BrAC_{<0.08}} \approx 50 \% + 100(\cdot5 - [Z_{0.08\rightarrow tail Table}]\% \]

\[ \approx 50 \% + 100(\cdot5 - 0.24) \% \]

\[ \approx 50 \% + 26 \% \]

\[ \approx 76 \% \]
Uncertainty From Breath to Blood

Breath as Indirect Measure of Blood
Conversion Factor

\[ BAC \, \frac{g}{100ml} = \frac{R}{2100} \cdot BrAC \, \frac{g}{210L} \]

Conversion Factor From Literature

\[ R_b = 2280 \]
\[ \mu_R = 265 \]

Best Estimate

\[ BAC_b = \frac{R_b}{2100} \cdot BrAC_b \]
\[ = \frac{2280}{2100} \cdot 0.0775 \]
\[ = 0.0841 \]

Combined Uncertainty

\[ \mu_{bl} = BAC_b \sqrt{\left(\frac{\mu_R}{R_b}\right)^2 + \left(\frac{\mu_c}{BrAC_b}\right)^2} \]
\[ = 0.0842 \sqrt{\left(\frac{265}{2280}\right)^2 + \left(\frac{0.00355}{0.0775}\right)^2} \]
\[ = 0.0842 \sqrt{0.01351 + 0.00210} \]
\[ = 0.01052 \]

Expanded Uncertainty

\[ U = 2.576 \cdot 0.01052 \]
\[ = .0271 \]
Result

\[ BAC = 0.0841 \pm 0.0271 \, \text{g} \quad \text{100ml} \quad \text{(~99%)} \]

Coverage Interval

\[ 0.0570 \, \text{g} \quad \text{100ml} \quad \rightarrow \quad BrAC_{\sim 99\%} \quad \rightarrow \quad 0.1112 \, \text{g} \quad \text{100ml} \]
**Probability “True” BAC is less than 0.08 g/100ml**

Step 1: Coverage interval.

\[ 0.0570 \leftrightarrow 0.1112 \]

Step 2: Bias corrected mean

\[
\text{BrAC}_c = \frac{b_l + b_u}{2} = \frac{0.0570 + 0.1112}{2} = 0.0841
\]

Step 3: Expanded uncertainty

\[
U = \text{BrAC}_c - b_l = 0.0841 - 0.0570 = 0.0271
\]

Step 4: Combined uncertainty

\[
\mu_c = \frac{U}{2.576} = \frac{0.0271}{2.576} = 0.01052
\]

Step 5: Z-factor

\[
Z_{\text{BrAC}_c \rightarrow 0.08} = \frac{\text{BrAC}_c - 0.08}{\mu_c} = \frac{0.0841 - 0.08}{0.01052} = 0.390
\]

Step 6: Probability associated with the z-factor

\[
P_{\text{BrAC} < 0.08} \approx 35\% [z_{0.390 \rightarrow \text{tail Table}}]
\]
DOCUMENTS & DATA
FOR EXERCISES
# DATAMASTER CALIBRATION CERTIFICATE

**Washington State Patrol Toxicology Laboratory Division**

**WSP LABORATORY:** 222 Tumwater Blvd. Bldg 16 Tumwater, WA 98504-2640  
**SERIAL NUMBER:** 140067  
**DATE COMPLETED:** 11/29/2010  
**CUSTOMER NAME:** Elma PD  
**CUSTOMER ADDRESS:** 124 North 3rd Street  
Check if calibration occurred at customer address: [ ] Elma, WA 98541

**CALIBRATION:**  
- As Found  
  - QAP Batch #: 10021  
  - Simulator #: G1886  
  - Sim Thermometer #: E953233  
  - Result (g/210L): .80

- Water  
  - Not Applicable  
- Ethanol  
  - .0021  
  - G1886  
  - B951237  
  - E953233

**CALIBRATION PROCEDURE USED:** WSP-TLD Technical Manual. Chapter 5.0 Quality Assurance Procedure  
**CALIBRATED ITEM:** NPAS DataMaster CDM  
**Serial Number:** 140067  
**Condition:** Good

## CERTIFICATION RESULTS (g/210L)

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| Mean  | 0.0400 | 0.0819 | 0.1003 | 0.1523 |
| SD    | 0.0005 | 0.0007 | 0.0007 | 0.0008 |
| Bias %| 1.27   | 3.02   | 5.0    | 2.35   |
| CV %  | 1.25   | 3.85   | 7.0    | 5.3    |

**COMPLETE BREATH TEST**  
- External Standard Batch #: 10039  
- Simulator #: DR4759  
- Sim Thermometer #: DR4759

**TECHNICIAN PERFORMING CALIBRATION**  
- Ruth Cramer  
- 11/29/2010  
- DATE CALIBRATION COMPLETED

**TECHNICIAN REVIEWING & ISSUING CERTIFICATE**  
- Danny Stumpf  
- Breath Test Technician  
- 11/29/2010  
- DATE ISSUED

**TRACEABILITY INFORMATION**  
This calibration is traceable to NIST through an unbroken chain of comparisons. The DataMaster/DataMaster CDM is calibrated using a QAP solution prepared by the Washington State Toxicology Laboratory. The QAP solution measurements are traceable to the results of the ethanol control CRM whose properties are traceable through its Certificate of Analysis to the NIST ethanol standard.

Temperatures are measured at 34.0±0.2°C using Guth model 34C or 2100 simulators equipped with mercury in glass thermometers or digital thermometers, respectively. Both types are certified against a Guth-Eutechics 4300 digital thermometer which is calibrated by ICL Calibration Laboratories that provides a certificate showing that the measurements are traceable to NIST.

Voltages are measured using a Fluke 70 III multi-meter which is calibrated by Fluke Corporation that provides a certificate showing that DC voltage measurements are traceable to an international voltage standard.

This certificate applies only to the item being calibrated and shall not be reproduced except in full, without the written approval of the WSP Toxicology Laboratory Division.

---

**TLD_DMCert**  
Revision: Original  
Approved by the State Toxicologist  
Effective Date: 06/11/09
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Analysis

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UNCERTAINTY IN MEASUREMENT: REQUIREMENT OF A COMPLETE RESULT (NOTES)


122: “The assessment of the accuracy of the conclusions from forensic analyses and the estimation of relevant error rates are key components of the mission of forensic science.”

116:

i. “Scientific data and processes are subject to a variety of sources of error...A key task...for the analyst applying a scientific method to conduct a particular analysis, is to identify as many sources of error as possible...and to estimate the magnitude of remaining errors so that the conclusions drawn...are valid. Numerical data reported in a scientific paper include not just a single value (point estimate) but also a range of plausible values (e.g., a confidence interval, or interval of uncertainty).”

ii. “As with all other scientific investigations, laboratory analyses conducted by forensic scientists are subject to measurement error. Such error reflects the intrinsic strengths and limitations of the particular scientific technique. For example, methods for measuring the level of blood alcohol in an individual or methods for measuring the heroin content of a sample can do so only within a confidence interval of possible values.”

117: “Consider, for example, a case in which an instrument (e.g., a breathalyzer such as Intoxilyzer) is used to measure the blood-alcohol level of an individual three times, and the three measurements are 0.08 percent, 0.09 percent, and 0.10 percent. The variability in the three measurements may arise from the internal components of the instrument, the different times and ways in which the measurements were taken, or a variety of other factors. These measured results need to be reported, along with a confidence interval that has a high probability of containing the true blood-alcohol level (e.g., the mean plus or minus two standard deviations). For this illustration, the average is 0.09 percent and the standard deviation is 0.01 percent; therefore, a two-standard-deviation confidence interval (0.07 percent, 0.11 percent) has a high probability of containing the person’s true blood-alcohol level. (Statistical models dictate the methods for generating such intervals in other circumstances so that they have a high probability of containing the true result.) The situation for assessing heroin content from a sample of white powder is similar, although the quantification and limits are not as broadly standardized.”

184: “Few forensic science methods have developed adequate measures of the accuracy of inferences made by forensic scientists. All results for every forensic science method should indicate the uncertainty in the measurements that are made, and studies must be conducted that enable the estimation of those values.”
There is a critical need in most fields of forensic science to raise the standards for reporting and testifying about the results of investigations.”

As a general matter, laboratory reports generated as the result of a scientific analysis...should identify, as appropriate, the sources of uncertainty in the procedures and conclusions along with estimates of their scale (to indicate the level of confidence in the results)...to allow the nonscientist reader to understand what has been done and permit informed, unbiased scrutiny of the conclusion.”

Forensic reports, and any courtroom testimony stemming from them, must include clear characterizations of the limitations of the analyses, including associated probabilities where possible.”

2. ISO, General requirements for the competence of testing and calibration laboratories, ISO 17025 (2005).

§ 5.4.6.2: “Testing laboratories shall have and shall apply procedures for estimating uncertainty of measurement. In certain cases the nature of the test method may preclude rigorous, metrologically and statistically valid, calculation of uncertainty of measurement. In these cases the laboratory shall at least attempt to identify all the components of uncertainty and make a reasonable estimation, and shall ensure that the form of reporting of the result does not give a wrong impression of the uncertainty…”

§ 5.10.3.1: “In addition to the requirements listed in 5.10.2, test reports shall, where necessary for the interpretation of the test results, include the following:...c) where applicable, a statement on the estimated uncertainty of measurement; information on uncertainty is needed in test reports when it is relevant to the validity or application of the test results, when a customer's instruction so requires, or when the uncertainty affects compliance to a specification limit;”


Knowledge of the uncertainty associated with measurement results is essential to the interpretation of the results. Without quantitative assessments of uncertainty, it is impossible to decide whether observed differences between results reflect more than experimental variability, whether test items comply with specifications, or whether laws based on limits have been broken. Without information on uncertainty, there is a risk of misinterpretation of results. Incorrect decisions taken on such a basis may result in unnecessary expenditure in industry, incorrect prosecution in law, or adverse health or social consequences.”

§ 5.1.1: “Measurement uncertainty relates to individual results. Repeatability, reproducibility, and bias, by contrast, relate to the performance of a measurement or testing process…”
4. **NIST, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST 1297 (1994).**

   **Forward:** “It is generally agreed that the usefulness of measurement results…is to a large extent determined by the quality of the statements of uncertainty that accompany them.”

   § 2.1: “In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement…and thus the result is complete only when accompanied by a quantitative statement of its uncertainty.”

   § 5.2: “It is assumed that a correction (or correction factor) is applied to compensate for each recognized systematic effect that significantly influences the measurement result and that every effort has been made to identify such effects.”

5. **JCGM, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM), (2008).**

   § 0.1, § 7.1.4: “When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its uncertainty”.

   § 1.1: “This Guide establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields — from the shop floor to fundamental research. Therefore, the principles of this Guide are intended to be applicable to a broad spectrum of measurements, including those required for:….complying with and enforcing laws and regulations…”

   § 1.4: “This Guide provides general rules for evaluating and expressing uncertainty in measurement rather than detailed, technology-specific instructions.”

   § 2.2.3: “Uncertainty (of measurement): Parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.”

   § 3.1.2: “In general, the result of a measurement is only an approximation or estimate of the value of the measurand and thus is complete only when accompanied by a statement of the uncertainty of that estimate.”

   § 3.2.4: “It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.”
§ 3.3.1: “The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand”

§ 3.3.7: “The intended purpose of $U$ is to provide an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.”

§ 3.4.7: “Blunders in recording or analysing data can introduce a significant unknown error in the result of a measurement. Large blunders can usually be identified by a proper review of the data; small ones could be masked by, or even appear as, random variations. Measures of uncertainty are not intended to account for such mistakes.”

Appendix D.5.2: Measurement uncertainty accounts for “the fact that, for a given measurand and a given result of measurement of it, there is not one value but an infinite number of values dispersed about the result that are consistent with all of the observations and data and one's knowledge of the physical world, and that with varying degrees of credibility can be attributed to the measurand.”


§1.1: “This Guide gives detailed guidance for the evaluation and expression of uncertainty in quantitative chemical analysis, based on the approach taken in the ISO “Guide to the Expression of Uncertainty in Measurement”. It is applicable at all levels of accuracy and in all fields - from routine analysis to basic research and to empirical and rational methods.”

§1.4: “It is assumed throughout this Guide that, whether carrying out measurements or assessing the performance of the measurement procedure, effective quality assurance and control measures are in place to ensure that the measurement process is stable and in control. Such measures normally include, for example, appropriately qualified staff, proper maintenance and calibration of equipment and reagents, use of appropriate reference standards, documented measurement procedures and use of appropriate check standards and control charts.”

§2.1.1: Uncertainty: “A parameter associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand.”

§2.1.2: “In many cases in chemical analysis, the measurand will be the concentration of an analyte.”

§2.4.14: “Uncertainties estimated using this guide are not intended to allow for the possibility of spurious errors/blunders.”

§6.7: Sources of uncertainty include: Sampling, Storage conditions, Instrument affects, Measurement conditions, Sample effects, Operator effects, Random effects.
§9.4.1: “Unless otherwise required, the result $x$ should be stated together with the expanded uncertainty $U$ calculated using a coverage factor $k=2$…The following form is recommended:


"(Result): ($x \pm U$)…using a coverage factor of 2, [which gives a level of confidence of approximately 95%]"

§B10 – Result of Measurement – Note 2: “A complete statement of the result of a measurement includes information about the uncertainty of measurement.”


§1: “In order to utilize a result to decide whether it indicates compliance or non-compliance with a specification, it is necessary to take into account the measurement uncertainty.”

8. EURACHEM - Compliance Leaflet Use of Uncertainty Information in Compliance Assessments.

Introduction: “When test results are used to assess compliance i.e. to decide whether specifications or regulations are met, the measurement uncertainty of the test results has to be taken into account.”

Example 2: “In law it is important not to punish an innocent person. The decision limit can be set to reduce the chance of this happening. Here is an example from measurement of blood alcohol (EtOH) in a sample taken from a driver in Sweden…”


ii: “Uncertainty of measurement is the most important single parameter that describes the quality of measurements. This is because uncertainty fundamentally affects the decisions that are based upon the measurement result.”

§ 1.1: The main purpose of measurement is to enable decisions to be made. The reliability of these decisions depends on knowing the uncertainty of the measurement results. If the uncertainty of measurements is underestimated, for example because the sampling is not taken into account, then erroneous decisions may be made that can have large financial consequences.”

§ 2.1: “The principles of this Guide are applicable to the estimation of uncertainty from the full range of materials that are subject to analytical measurement (e.g. gaseous, liquid and solid). These include…forensic materials…”
§ 6.1: “There are two broad approaches to the estimation of uncertainty."

i. “One of them, described as ‘empirical’, ‘experimental’, ‘retrospective’, or ‘top-down’, uses some level of replication of the whole measurement procedure to give a direct estimate of the uncertainty for the final result of the measurement. This approach is called the ‘empirical’ approach in this Guide.”

ii. “The second, variously described as ‘modelling’, ‘theoretical’, ‘predictive’ or ‘bottom-up’, aims to quantify all of the sources of uncertainty individually, and then uses a model to combine them. It will accordingly be referred to as the ‘modelling’ approach.”

iii. “These approaches are not mutually exclusive. The empirical method can be adapted to estimate contributions to uncertainty from one or more effects or classes of effect. Both approaches can usefully be used together to study the same measurement system, if required. The applicability of the two approaches varies between the different materials to be sampled.”

§ 9.1.1: “The empirical (‘top-down’) approach is intended to obtain a reliable estimate of the uncertainty, without necessarily knowing any of the sources individually. It relies on overall reproducibility estimates from either in-house or inter-organisational measurement trials.”

§ 10.1.1: “The modelling approach, often colloquially known as ‘bottom-up’, has been described for measurement methods in general [2], and applied to analytical measurements [1]. It initially identifies all of the sources of uncertainty, quantifies the contributions from each source, and then combines all of the contributions, as a budget, to give an estimate of the combined standard uncertainty.”

§ 12.1: “The empirical (top-down) and modelling (bottom-up) approaches each have their advantages in certain circumstances.”

§ 14.6.1:

i. “Results are often compared with tolerances or regulatory limits in order to assess compliance with a requirement. In making such comparisons, it is important to take uncertainty into account…The basic principles are:…For proof of non-compliance, the result and its uncertainty interval must be entirely outside the permitted range.”

ii. “Criminal prosecution in most countries, however, requires clear proof of non-compliance and in these circumstances (e.g. blood alcohol prosecutions) it is normal practice to seek proof of non-compliance at high levels of confidence.”

§ 1.0:

i. “Given the inherent variability of measurement, a statement of a measurement result is incomplete (perhaps even meaningless) without an accompanying statement of the estimated uncertainty of measurement (a parameter characterizing the range of values within which the value of the measurand can be said to lie within a specified level of confidence).”

ii. “The ISO *Guide to the Expression of Uncertainty in Measurement* (GUM)…provide the current international consensus method for estimating measurement uncertainty. It is equally applicable to calibration and test results and it forms the basis for accreditation requirements relating to measurement uncertainty estimation.”

§ 3.8: “When reporting the result of a measurement, at a minimum one should

i. Give a full description of how the measurand $\tilde{Y}$ is defined;

ii. State the result of the measurement as $Y = y \pm U$ and give the units of $y$ and $U$;

iii. Give the value of the coverage factor $k$ used to obtain $U$;

iv. Give the approximate level of confidence associated with the interval $y \pm U$ and state how it was determined.”

11. **A2LA Specific Requirements: Forensic Examination Accreditation Program – Testing (2010).**

§ 5.10 F1.3: “Reports produced by the forensic organization shall include a description of the error rate, measurement uncertainty or uncertainty of the determination where available and in accordance with written guidelines.”

12. **NATA Assessment of Uncertainties of Measurement for calibration & testing laboratories (2002).**

8: “Every measurement made has an error associated with it, and, without a quantitative statement of the error, a measurement lacks worth. Indeed without such a statement it lacks credibility. The parameter that quantifies the boundaries of the error of a measurement is called the uncertainty of measurement.”

13. **NATA - Uncertainty Of Measurement In Biological, Forensic, Medical And Veterinary Testing (2003).**

3: “NATA does not prescribe a particular approach to estimating uncertainty of measurement. At assessment, laboratories will be required to justify their chosen approach and in accordance with Clause 5.4.6, must use an approach that produces a “reasonable estimate”.”
APPENDIX 2: Quantitative tests in forensic science (to which uncertainty applies).

i. “Drugs”
ii. “Blood alcohol”
iii. “Breath alcohol measurement”
iv. “Drugs in drivers”
v. “Toxicology”


4: “Knowledge of the uncertainty of measurement of testing results is fundamentally important for laboratories, their clients and all institutions using these results for comparative purposes.”

6: “According to ISO/IEC 17025, testing laboratories must report uncertainty estimates where specified by the method, where required by the client and/or where the interpretation of the result could be compromised by a lack of knowledge of the uncertainty. This should at least be the case where testing results have to be compared to other testing results or other numerical values, such as specifications.”

15. **Gullberg, Statistical Applications in Forensic Toxicology, Medical-Legal Aspects of Alcohol, 457 (James Garriott ed., 5th ed. 2009).**

458: “Many would consider inadequate statistical thought in experimental design and data analysis to be unethical scientific practice.”

496: “measurement results should be corrected for any known bias.”

504:

i. “Communicating analytical results occurs during the post-analytical stage of a complete measurement process. No important measurement process is complete until the results have been clearly communicated to and understood by the appropriate decision maker. Forensic measurements are made for important reasons. People, often unfamiliar with analytical concepts, will be making important decisions based on these results. Part of the forensic toxicologist’s responsibility is to communicate the best measurement estimate along with its uncertainty. Insufficient communication and interpretation of measurement results can introduce more uncertainty than the analytical process itself. The best instrumentation along with the most credible protocols ensuring the highest possible quality control will not compensate for the unclear and insufficient communication of measurement results and their significance.”

ii. “Clear and sufficient communication of measurement results begins with adequate printed documentation. Measurement results and associated information read by
decision makers should be clear, thorough and self-explanatory. The results must display...the associated uncertainty of the results. The uncertainty estimate can take the form of a...expanded uncertainty or a confidence interval...whenever possible, a numerical assessment of uncertainty should be provided.”


562: “Figure 1 illustrates the importance of considering uncertainty estimates in the context of legally prohibited breath alcohol limits. Clearly, given a 0.08 g/210 L limit, the court should be informed if the uncertainty in case B holds rather than that of A. Once the acceptable uncertainty and fitness-for-purpose is established, the court can appropriately weigh the evidence and make an informed decision.”

563:

i. “bias can be corrected [and] the combined uncertainty…easily determined with standard statistical methods.”

ii. “The legal admission of forensic breath-test results is rarely accompanied by an estimation of its uncertainty. This results, in part, from final decision-makers failing to appreciate its relevance. Defense attorneys, prosecutors, judges and lay juries often lack scientific training and naively accept measurement results as certain.”

iii. “Moreover, forensic scientists themselves often fail to consider or appreciate measurement uncertainty.”

iv. “Although some forensic scientists may find the notion of ‘error’ unsettling, it is a reality of measurement that must be appreciated...Only when measurement ‘error’ is acknowledged and properly estimated can...analytical goals [be] achieved.”

33:

i. Abstract. Widmark’s “equation is employed to estimate either the number of drinks consumed or the corresponding blood or breath alcohol concentration. Despite the wide use of Widmark’s equation, rarely is an uncertainty estimate also provided…Including valid estimates of uncertainty should enhance the legal admissibility and confidence for Widmark estimations.”

ii. “Despite the wide application of Widmark’s equation in many contexts today, there seems to be little appreciation for its uncertainty…The only forensically appropriate way to present and interpret Widmark estimates is to include an assessment of their uncertainty. Failing to acknowledge uncertainty is probably most pronounced in the courts where juries are asked to consider and weigh the quantitative estimates. Often lacking an appreciation for quantitative uncertainty, juries tend to assign an unmerited amount of weight to the estimates. Forensic scientists, therefore, should be prepared to present a reliable estimate of uncertainty along with any Widmark estimates.”


25: “Results of scientific measurements are compelling to those untrained in numerical or analytical issues while many believe that all numerical results possess absolute certainty. The professional expert witness, however, must present numerical information accompanied by their limitation and avoid conveying the “illusion of certainty”. The misuse and misleading application of statistics, designed to convey an unjustified interpretation, must also be considered unethical. Doubt and uncertainty should be respectable concepts in the forensic sciences. While fitness-for-purpose can and should certainly be established, assumptions and uncertainty in breath alcohol analysis must be acknowledged.”


93 II.A:

i. “Measurement uncertainty near the critical limit is a fair and relevant argument. Breath alcohol analysis results, like all measurements, possess uncertainty. Forensic scientists must be prepared to acknowledge this and compute appropriate estimates. Figure 1 illustrates this issue by showing different hypothetical uncertainty intervals (A and B) that can arise in different measurement contexts for the same sample mean (0.088 g/210L). This could be applied to any critical concentration. The question is, which uncertainty interval is correct for a particular case? Clearly, this would be relevant for the court to know…”
ii. “…Clearly, at some mean BrAC, the lower 95%, 99%, or other selected confidence interval limit will fall below the critical level. This reality of measurement must be acknowledged.”

iii. “The approach taken by some European jurisdictions is to perform this calculation and then offer evidence in the prosecution only of the lower 99% confidence interval limit exceeds the statutory limit.”

94: “Preparation prior to trial is very important on this issue. The forensic scientist must have the relevant information and perform the computations before trial. These must also be disclosed to attorneys for both sides prior to trial so that all are aware of the computation along with their assumptions and limitations.”


30: i. “Breath alcohol measurement has variability resulting from instrumental, procedural and biological components. Reliable estimates of the standard deviation (S.D.) are necessary for calculating uncertainty in the form of confidence intervals.”

ii. “The presentation of breath alcohol results in drunk driving trials should ideally be accompanied by an estimate of their uncertainty. This estimate is generally in the form of a standard deviation (S.D.) which can subsequently be used to determine a confidence interval for the person’s mean breath alcohol concentration (BrAC). A confidence interval is an intuitive concept while providing the court with relevant information to…assist in weighing the breath alcohol evidence in view of the appropriate per se statutory limit (e.g. 0.08 g/210 l in many jurisdictions.”


50: “All analytical results, regardless of context, protocol or instrumentation, possess uncertainty…all measurement results are approximations. This is acceptable…so long as the limits of uncertainty are known and acceptable.”

§VI (p.60): “All analytical results, including breath alcohol analysis, have uncertainty.”

61: Confidence intervals
“Employing the same fundamental equation for a confidence interval, the probability that an individual’s mean breath alcohol concentration exceeds some critical level can also be determined.”

22. **Gullberg, Considering Measurement. Variability When Performing Retrograde Extrapolation of Breath Alcohol Results (1994).**

126:

i. “In most cases, the known measurement result is assumed to be the beginning point without any regard to its uncertainty or variability. It should not be assumed that the BrAC measurement is a fixed constant with no uncertainty. In fact, a person’s BrAC is reasonably considered a continuous random variable sampled at one point in time from a normally distributed population of values.”

ii. “The mean should also be corrected for any known systematic error or bias.”

127: “Finally, BrAC measurements should be considered random variables drawn from assumed normal distributions possessing variability.”


247: “Random error is associated with breath alcohol measurements, as with all analytical methods. The total random uncertainty of a group of n measurements is typically determined by computing the standard deviation...The total random uncertainty has two primary sources: the instrumental method and the sample source...In breath alcohol testing the two primary sample sources are simulators and human breath.”

24. **A.W. Jones, Ph.D, Dealing with Uncertainty in Chemical Measurements, 14(1) Newsletter of the International Association for Chemical Testing, 6 (2003).**

6:

i. “The results generated by all analytical methods are subject to uncertainty and this also applies to the determination of ethanol in blood, breath or urine for legal purposes.”

ii. “The magnitude of uncertainty becomes important when the test result is compared with some reference point or threshold value and when a decision is made whether or not the result exceeds the critical threshold.”
iii. “When per se DUI, DWI or OUI laws are enforced in most European countries some kind of allowance for uncertainty is always made.”

iv. “When the first alcohol per se drunk driving law was introduced in Sweden in 1941…the Supreme Court mandated that the laboratory charged with the task of analyzing the blood samples should allow for uncertainty or error in the analytical procedures. The forensic chemistry government laboratory therefore from the very beginning always made a deduction from the mean result of analysis.”

7: 

i. “An urgent need exists to report results of forensic alcohol analysis as a range of values, that is as a confidence statement.”

ii. “Alternatively the laboratory responsible for the analysis could be charged with making a deduction for uncertainty from the average of duplicate tests.”

10: 

i. “If systematic error does exist this must be added or subtracted from the mean result of alcohol analysis before the uncertainty calculations are made.”

ii. “the software can be programmed to incorporate these calculations and print the amount deducted along with the final result.”


1116: 

i. “This paper describes the analysis of ethanol in blood specimens from suspect drunk drivers and the associated quality assurance procedures currently used in Sweden for legal purposes…A deduction is made from the mean analytical result to compensate for random and systematic errors inherent in the method…Accordingly, the reduced prosecution BAC is less than the actual BAC with a statistical confidence of 99.9%.”

ii. “The result used for prosecution must not exceed the true value with at least 99.9% confidence.”

1120: 

i. “Statistical Background” – Describes error approach to determining level of confidence of blood test results.
ii. “Results” – Describes error approach to determining level of confidence of blood test results.

1125: “The notion of making an allowance to adjust the mean result of analysis for errors in the method is a long-established tradition in Sweden. This is especially relevant when per se statutes are enforced and the the BAC results cannot be rebutted.”

1126: “…reducing the mean result so that the final prosecution value is less than the true BAC with high statistical confidence such as 99.9%...as used for legal purposes in Sweden.”


456: “[E]ven a result from a well-controlled method inevitably suffers from uncertainty.”

456-457: The determination of uncertainty in blood alcohol measurements adheres to the same methodology recognized in other areas of science following the rules given by the GUM.

463: “It should be emphasized that only the combined standard uncertainty should be used to establish such safety margins. The analytical uncertainty is a part of the combined standard uncertainty of measurement; hence, basing the safety margin on the analytical uncertainty alone will overestimate the safety provided by it.”


207: “The accuracy of measuring this alcohol concentration is obviously of prime concern as an erroneous result can avert the administration of justice. The common practice [in Hong Kong China] is to deduct all errors from the measured value and compare the deducted value with the prescribed limit, so that the benefit of all errors of the measurement is given to the driver. It is therefore important for any laboratory responsible for measuring blood alcohol concentrations to identify and quantify all errors associated with the measurement.”

767:  
i. “This article illustrates how proficiency test results provide the basis for estimating uncertainties in three instances: (i) For breath alcohol analyzers…(ii) For blood alcohol…”

ii. “This article illustrates a simple and reliable approach for estimating uncertainties…It is applicable to a wide range of chemical test results reported by forensic alcohol and toxicology laboratories.”

iii. “Interlaboratory comparisons have long been recognized in the literature as an important means for estimating the range of errors (i.e., the UM) associated with a chemical analysis.”

iv. “This article illustrates three similar approaches for applying proficiency test data to the estimation of uncertainties for common assays: breath alcohol calibrations, blood alcohol determinations, and forensic toxicology (i.e., drug) quantitations. In each case, we will summarize the available data, explain the approach, and identify major assumptions and limitations.”

767: Breath alcohol  
769: Blood alcohol  
770: Toxicology

29. **King - International interlaboratory study of forensic ethanol standards, 124 ANALYST 1123 (1999).**

1123:  
i. “Ethanol in blood and urine analyses are routinely undertaken in both clinical and forensic laboratories, in connection with drinking and driving legislation, for example, under the UK Road Traffic Act 1981…Prosecution only takes place when the measured level exceeds the legal limit by a margin which aims to take account of the measurement uncertainty. Current best practice relies on method validation to establish that the method employed is free from bias and uses precision data to establish 99% confidence intervals.”
“This paper reports the study protocol and the results of the interlaboratory comparison and discusses the results in the context of the metrological performance required to support routine measurement used for prosecution purposes. Measurement uncertainty estimates were made using the method described in the Eurachem Guide, which also provides a general introduction to the subject.”

1127: “…the situation in the UK is that while the legal limit for a driver’s blood ethanol level is 80 mg per 100 ml, prosecutions are only initiated when the measured level exceeds 87 mg per 100 ml. Thus, to enforce a legal limit of 80 mg per 100 ml by prosecuting at 87 mg per 100 ml, the standard uncertainty of the result should be 52.3 mg per 100 ml (or 2.9% of the legal limit) for a confidence level of 99%. The measured value of 87 mg per 100 ml can then be held, with 99% confidence, to exceed 80 mg per 100 ml, as the measured value exceeds the legal limit by three standard deviations.”

1130: “2. It is possible to calculate the acceptable uncertainty of ethanol in blood measurements required to enable reliable decisions to be made concerning compliance with legal limits for drink-driving.”

30. **Christensen, Rules for stating when a limiting value is exceeded** 7 **ACCRED. QUAL. ASSUR.** 28 (2002).

28: “The paper describes rules for stating whether a measurement result indicates that the value of the measurand, e.g. the concentration of a substance in the blood, is in conformity or not in conformity with given specifications. Examples of the intended applications are…alcohol level in drivers’ blood…”

29:
   i. “It is important to stress that the uncertainty of measurement has to be taken into account when testing conformity with specification limits.”
   ii. Example: “The LV for alcohol in drivers’ blood is 0.2‰, in most countries.”

30: “When comparing results of measurement with LVs, it is necessary to give quantitative indication of the uncertainty of measurement.”

“Chromatographic techniques are very frequently used in analytical procedures for the separation, determination and identification of a wide spectrum of analytes present in samples with complex and sometimes variable matrices. However, the estimation of uncertainty of the final results does not include the uncertainties associated with the actual chromatographic process. In effect, such results cannot always be treated as a reliable source of analytical information. In this paper we present the basic terms, sources of uncertainty, and methods of calculating the combined uncertainty that any presentation of final determinations should include.”

“Uncertainty is a basic characteristic of any measurement; uncertainty is always present, at every step of a procedure.”

“The systematic error is responsible for the accuracy of the final determination, and its value should be calculated during the validation of the analytical procedure. The result of the final determination can then be corrected using the calculated systematic error.”

“Random errors are the cause of uncertainty associated with the course of the analytical process and the plot of measurement results. This type of error should be regarded as a random variable (hence its name); thus, the final determination should always be treated as an approximation (estimate) of the true value.”

“Every analytical result is associated with uncertainty (for the sources of uncertainty – see Table 2). Therefore, the uncertainty of the result of a determination must be calculated and accompany its presentation. Moreover, an analytical result must be recorded not as one value, but according to the values of a continuous random variable, as a confidence interval, i.e. the interval likely to include the expected value. Here is an example of the correct presentation of an analytical result:

\[ \text{CPCB-28} \pm U \ (k = 2) = 31.3 \pm 2.7 \text{ngg}^{-1} \]

where PCB-28 is the analyte concentration (here an analyte from the PCB group – PCB-28 according to IUPAC) calculated as the mean of a series of parallel determinations; \( U \) is the expanded uncertainty of the measurement and \( k \) is the coverage factor (for \( P = 95\%, \ k = 2 \).)”
Trial by Numbers

Uncertainty in the Quest For Truth and Justice

“All results for every forensic science method should indicate the uncertainty in the measurements that are made, and studies must be conducted that enable the estimation of those values.”

On Aug. 5, 2010, prosecution expert Rod Gullberg was handed a breath alcohol test ticket with the values 0.081 and 0.080 printed on it. Assuming the lab followed proper quality assurance procedures and testing protocols, all parties agreed that these were the results of an accurate and reliable test. Gullberg was then asked, given these results and the fact that this was an accurate and reliable test, could he state beyond a reasonable doubt that this individual’s breath alcohol concentration (BAC) exceeded a 0.080 (the per se limit in the state of Washington). Gullberg responded: “I would have to say yes based on these results here.”

Similar evidence and testimony, concerning a range of forensic measurements, are introduced in courtrooms around the country every day. And based on such evidence and testimony, citizens accused of all manner of crimes are found guilty. In the context of a prosecution for driving under the influence of alcohol, where guilt may be based on a number alone and a machine is the only way to determine an individual’s breath or blood alcohol concentration, many simply plead guilty in the face of such evidence. But what if the results from an accurate and reliable test do not actually mean what most of us presume?

Despite the fact that the test under consideration was agreed to be accurate and reliable, within 10 minutes of his testimony Gullberg reversed himself, stating that he could not conclude based on the test results that the individual’s BAC was in excess of a 0.080. In fact, he conceded that while the test...
was accurate and reliable, there was actually a 44 percent likelihood that the individual’s BAC was below a 0.080! Far more than a reasonable doubt, these “accurate and reliable” test results barely established the conclusion as more likely than not! Absent those critical 10 minutes, an innocent citizen could have been convicted based on evidence that meant something very different from what the state presented it to establish.

What happened in those 10 minutes to change Gullberg’s opinion? Did he lie? Were the wrong values printed on the breath test ticket? Was there something wrong with the test?

Measurement Uncertainty

To many, the result of a measurement represents a singular, well-defined property of a thing being measured (the “measurand”). In such a world, a breath test result of 0.080 would be interpreted as representing an individual’s true and specific breath alcohol concentration.\(^2\) (See Figure 1.)

Unfortunately, reality is not quite so simple. For even the most carefully performed measurement, the value of a thing being measured can never be known exactly; all that can ever be given is an estimated value.\(^3\)

Thus, in the real world, a breath test result of 0.080 is more appropriately represented as a packet of values, any of which could actually be attributed to an individual’s BAC. (See Figure 2.)

If the illustration in Figure 2 is reminiscent of the familiar Bell Curve, it is no coincidence. The information obtained from a measurement, which we call its result, is actually a probability distribution that characterizes our knowledge of the measured quantity.\(^3\) That we can never know the singular true value of the thing being measured is due to many factors including “measurement error” and imperfect information concerning the measuring system and thing to be measured.

Measurement uncertainty “reflects the lack of exact knowledge of the value of the measurand.”\(^6\) It provides a quantitative statement characterizing the dispersion of values that can actually and “reasonably be attributed to the measurand.”\(^7\) It is well-recognized that “the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement and thus the result is complete only when accompanied by a quantitative statement of its uncertainty.”\(^8\)

For example, “[n]umerical data reported in a scientific paper include not just a single value (point estimate) but also a range of plausible values (e.g., a confidence interval, or interval of uncertainty).”\(^9\)

The most common way of expressing measurement uncertainty is as a coverage interval. It consists of a range of values that can be attributed to the measurand as well as a level of confidence that the “true” value is contained within that range. Assuming a measured value of \(y\) and an expanded uncertainty \(U\) determined to have a 95 percent likelihood of containing the true value of a measurand, a complete measurement result \(X\) and the accompanying coverage interval would be expressed as follows:

\[
\text{Measurement Result} = \text{Value} \pm \text{Uncertainty}
\]

\[
X = y \pm U \ (95\%)
\]

\[
\text{Coverage Interval}
\]

\[
y - U \leq X \leq y + U
\]

Returning to the example of a breath test result of 0.080, and assuming an uncertainty of ± 0.010 with a 95 percent level of confidence, the right and wrong way to conceive of and report the result of the BAC measurement is shown in Figure 3.

Thus, despite the fact that the value reported is a 0.080, all we can really say is that the values that can actually be attributed to the BAC in question range from 0.065 to 0.095.

\[
\text{Test Value: } 0.080
\]

\[
\text{BAC} = 0.080 \pm 0.010 \ (95\%)
\]

\[
\text{Test Value: } 0.080
\]

\[
\text{BAC} = 0.080
\]
from .070 to .090 with a 95 percent level of confidence. This applies to all forensic measurements. Whether it is measuring the level of blood alcohol in an individual, the heroin content of a sample or any other quantity subject to measurement, the quantities of interest can be determined “only within a confidence interval of possible values.”

Although there are different approaches for determining uncertainty, the same general principles and tools utilized are applicable to all measurements. First, all sources of uncertainty that may affect the use to which the result is put must be taken into account. A common way to document sources of measurement uncertainty, as well as their relationship to each other and the final result, is a cause and effect diagram. (See Figure 4.)

Once the relevant sources of uncertainty have been identified, the amount of uncertainty contributed by each must be determined. These values are then added together to yield the combined uncertainty, $U_c$. Multiplying the combined uncertainty by an appropriate coverage factor, $k$, generates the expanded uncertainty, $U = kU_c$, discussed above. This information is commonly documented in an uncertainty budget. (See Figure 5.)

The coverage factor, shown in Figure 6, is important because it determines how large the coverage interval will be and the level of confidence associated with it. The actual level of confidence associated with a given coverage factor depends upon the probability distribution associated with the measurement. For most real world situations, the underlying distribution will be approximately normal so that $k = 2$ yields a level of confidence of approximately 95 percent and $k = 2.576$ gives a level of confidence of approximately 99 percent.

**Coverage Interval**

$$y - k\mu_c \leq X \leq y + k\mu_c$$

One important thing to note is that the uncertainty associated with a measurement is likely to differ when the measurement comes from two different sources. Accordingly, even where two measurements from distinct entities report identical values, the results may have very different meanings. For example, assume two individuals submit to a breath test but on different breath test machines, and that each test yields a value of 0.095. Given that the uncertainties associated with each test are likely different, the values reported may give a clear indication that one of these individual's BAC is over a 0.08 while revealing that the values that could actually and reasonably be attributed to the other's BAC include those under the per se threshold. (See Figure 7.)

Here, identical test values but with different uncertainties yield different results and different interpretations. Depending on which circumstance applies, a jury may come to a very different conclusion. Clearly, "considering or not the uncertainty of a critical
result can make the difference between acquittal and a guilty sentence.”

Again, the same thing applies to all measurements, not just those pertaining to forensic alcohol analysis.

Knowledge of the uncertainty associated with measurement results is essential to the interpretation of the results. Without quantitative assessments of uncertainty, it is impossible to decide whether observed differences between results reflect more than experimental variability, whether test items comply with specifications, or whether laws based on limits have been broken. Without information on uncertainty, there is a risk of misinterpretation of results. Incorrect decisions taken on such a basis may result in unnecessary expenditure in industry, incorrect prosecution in law, or adverse health or social consequences.

Measurement uncertainty is “fundamental to the interpretation and reporting of results.” Absent a statement of uncertainty, a result “lacks worth [and] credibility” and may be considered “meaningless.” In particular, “[a]ll results for every forensic science method should indicate the uncertainty in the measurements that are made.” When the result of a forensic measurement is reported simply as “a number,” it does not reflect the accuracy of the measurement and cannot be properly interpreted. “Estimating and reporting measurement uncertainty with the number completes the picture and allows us to properly use the result to make reliable and defensible decisions.”

Some Answers

What happened in those critical 10 minutes to change Rod Gullberg’s opinion? When he was initially presented with the “results” of the breath test in question, they were incomplete because they did not include any information concerning their uncertainty. (See Figure 8.) As already shown, the picture created by such incomplete results is rather simplistic. (See Figure 9.) Without more information, the breath test ticket clearly seems to communicate that the BAC of the individual in question exceeded the legal limit.

It was only after Gullberg had declared that he could conclude that this individual’s BAC exceeded a 0.080 beyond a reasonable doubt that he was provided with the test’s uncertainty. To a 99 percent level of confidence, the coverage interval was defined as 0.0731 to 0.0877. That means the values that could actually and reasonably be attributed to the BAC in question ranged from 0.0731 to 0.0877 with a 99 percent level of confidence. This creates a very different picture indeed.

In fact, by visual inspection alone...
(Figures 10 and 11) we can determine that it is almost as likely that this individual’s actual BAC is under the legal limit as it is over.

And given the coverage interval, Gullberg was easily able to confirm that the likelihood that this individual’s true BAC was under a 0.080 was 44 percent. To understand how this could be determined from the coverage interval, remember that the test result, and hence the coverage interval itself, is characterized by a Gaussian probability distribution, i.e., the Bell Curve. (See Figure 12.)

If the total area under the Bell Curve is defined so as to equal 1, the probability that the result lies within any range of values is simply given by the area under the curve contained within that range. Hence, the probability that this particular BAC was actually less than the legal limit is given by the area under the curve within the range from 0.0 to 0.079. (See Figure 13.)

At this point it should be recognized that the inclusion of uncertainty is not a “get out of jail free” card for those charged with DUI or any other crime. Just as the uncertainty may demonstrate a high likelihood that an individual with test values above the legal limit is actually below that limit, it can go the other way as well. It may show that there is a high likelihood that an individual with test values below the limit is actually above that limit. In general, the uncertainty favors neither party. It simply facilitates the discovery of truth by enabling proper interpretation of the evidence. Moreover, except in those cases where the evidence of guilt consists solely of a measurement result, measurement uncertainty does not dictate a particular outcome. Although necessary for the proper interpretation of a measurement result, it is simply another piece of the evidence for the jury to consider and weigh with the rest of the evidence in arriving at a verdict.

Rod Gullberg did not lie. The wrong values were not printed on the breath test ticket. There was nothing wrong with the test. Gullberg simply had not been provided sufficient information upon which to base a reliable and defensible opinion. State Toxicologist Fiona Couper and Quality Assurance Manager Jason Sklerov faced similar lines of questions. Predictably, they were also unable to properly interpret the state’s breath test results absent information concerning each test’s measurement uncertainty. Each of the state of Washington’s top three experts had been asked to interpret the results of breath tests obtained by their own program. And each was unable to do so absent information concerning each test’s uncertainty.

**Uncertainty in the Quest For Truth and Justice**

The aforementioned testimony was obtained during a week-long evidentiary hearing before a panel of three King County District Court judges. The primary subject of the hearings was whether the state could offer breath test results as evidence in prosecutions for
DUI without providing both the defendant and jury the uncertainty associated with those results.

Six months earlier, a similar question was raised before Commissioner Paul Moon of the Snohomish County District Court with respect to the admissibility of a blood test result absent its uncertainty. The commissioner found the blood test inadmissible under Washington Rules of Evidence 702 and 403. With respect to the first evidentiary provision, the court found:

If an expert testifies that a particular blood alcohol content measurement is value \( A \), without stating a confidence level, it is this court’s opinion that the evidence is being represented as an exact value to the trier of fact … [and] that presenting to the trier of fact the result of a blood test as an exact numerical value without stating a confidence level, is not generally acceptable in the scientific community and misrepresents the facts to the trier of fact. … This court holds that the result of the blood test in this case is not admissible under ER 702 in the absence of a scientifically determined confidence level because it misrepresents the facts and therefore cannot be helpful to the trier of fact.

Addressing Evidentiary Rule 403, the court explained:

It has been this court’s experience since 1983 that juries it has presided over place heavy emphasis on the numerical value of blood alcohol tests. To allow the test value into evidence without stating a confidence level violates ER 403. The probative value of this evidence is substantially outweighed by its prejudicial value. Therefore this court holds that the result of the blood test in this case is not admissible under ER 403 in the absence of a scientifically determined confidence level.

The prosecution chose not to present any witnesses at this earlier proceeding. With this as prologue, however, the prosecution presented testimony from the state’s three top breath test experts at the King County hearings. These experts proved of little benefit to the prosecution.

King County prosecutors were forced to acknowledge that their own experts were unable to properly interpret the breath test results presented absent information concerning each test’s uncertainty. They also acknowledged that it was unlikely that the typical defendant or juror would fare any better and may be misled by such results as easily as the prosecution’s experts were. Nonetheless, the state argued that it had no duty to provide the uncertainty of breath test results to either the defendant or jury, and that the court had no power to require it to do so. It maintained that even though it knew that its evidence was incomplete and subject to being misleading and misinterpreted when unaccompanied by measurement uncertainty, the justice system was intended to permit whatever results such evidence might engender — even if it meant that innocent citizens would be deprived of their liberty and guilty individuals set free as a result.

Washington prosecutors are not alone in this mindset. Although a few forensic labs properly account for uncertainty in the results they report, "most [forensic] reports do not discuss measurement uncertainties or confidence limits." Yet it is exactly this type of incomplete and often misleading evidence that is offered by prosecutors around the country every day. What is more alarming is that courts around the country permit this very evidence to form the basis for depriving citizens of their liberty on a daily basis as well. Such practices not only threaten individual liberty, but strike at the integrity of the justice system itself by hindering its ultimate mission of determining the truth. As the King County Court noted:

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A prosecutor is a participant in a system of criminal justice which is, by design, adversarial. Yet, a good prosecutor will never let the desire to “win” overcome his or her sense of justice. A trial court will follow precedent when it rules on matters before the court, but precedent will never be allowed to overcome the determination of a good judge to do justice in each and every case. What was trustworthy and reliable yesterday may not be today. As concepts of justice advance through each generation of police, criminal justice practitioners, attorneys and judges, we aim to provide better justice than was provided by those before us. As concepts of science change, we also need to be ready to move forward with those new, better practices. Nor should the court allow an instrument or a machine to determine an element of a criminal offense — unless there are appropriate safeguards to ensure that the evidence provided by the machine is what it purports to be. It bears repeating that these safeguards are foundational to our criminal justice system.

In the end, what this issue boils down to is plain and simple truth. The defense in this hearing was not asking for something that would derail prosecutions or preclude convictions. It was simply asking the court to require the state to report the results of its forensic measurements in a complete and accurate manner so that both defendants and jurors could properly interpret that evidence and would not be misled by it. The court saw the issue the same way:

> When a witness is sworn in, he or she most often swears to “tell the truth, the whole truth, and nothing but the truth.” In other words, a witness may make a statement that is true, as far as it goes. Yet there is often more information known to the witness, which if provided, would tend to change the impact of the information already provided. Such is the case when the state presents a breath-alcohol reading without revealing the whole truth about it. That whole truth, of course, is that the reading is only a “best estimate” of a defendant's breath-alcohol content. The true measurement is always the measurement coupled with its uncertainty.

The court subsequently recognized that “a breath-alcohol measurement without a confidence interval is inherently misleading.”

Neither the lab nor the prosecution provided the court with any reason why uncertainty either was not or could not be provided with the result of every test. In Washington, the uncertainty of every breath test that will be conducted on an instrument over the course of a year can be determined in five minutes at the time of the instrument’s annual calibration using an Excel spreadsheet. Thus, whether it is one test, 100 tests, 1000 or tens of thousands, the uncertainty of all these tests together can be determined in five minutes, once a year, and then printed up in a table to be supplied to every defendant and jury along with the test results. Given the ease with which the uncertainty can be determined and supplied, one is left wondering why the state would not want to supply this information.

The panel concluded that for breath test results to be admissible in prosecutions for DUI, both the defendant and jury must be provided with the uncertainty associated with those results. First, under principles of Due Process and the rules governing discovery, it stated:

> [W]e now place the state on notice that every discovery packet supplied to defendants must contain the confidence interval for any breath-alcohol measurement the state intends to offer into evidence in that case. Should the state fail to comply with this discovery order, then upon objection, such breath-alcohol measurement will not be admitted at trial.

Then, under Evidentiary Rule 702, the court found:

> Once a person is able to see a confidence interval along with a breath-alcohol measurement, it becomes clear that all breath-alcohol tests (without a confidence interval) are only presumptive tests. The presumption, of course, is that a breath-alcohol reading is the mean of two breath samples. This answer, however, is obviously incomplete. (Put another way, a breath-alcohol measurement without an uncertainty measurement does not tell the “whole truth.” RCW 5.28.020.) As discussed above, a breath test reading is only a “best estimate” of an individual's breath-alcohol level. The determination of a confidence interval completes the evidence. Therefore, upon objection, a breath-alcohol measurement will not be admitted absent its uncertainty level, presented as a confidence interval.

Thomas Bohan, immediate past president of the American Academy of Forensic Sciences, hailed the King County Court opinion as a landmark decision, engendering a huge advance toward rationality in our justice system and a victory for both forensic science and the pursuit of truth.

**Conclusion**

“The ultimate mission of the system upon which we rely to protect the liberty of the accused as well as the welfare of society is to ascertain the factual truth.” Complete, competent, and impartial forensic science investigations can be that ‘touchstone of truth’ in a judicial process that works to see that the guilty are punished and the innocent are exonerated.”

Given the potential consequences to individuals and society alike, however, reliance upon forensic science “is not a matter to take lightly, or to trust to luck.” Accordingly, “[i]n this age of science we must build legal foundations that are sound in science as well as in law.” This can be achieved “only by requiring scientific evidence to conform to the standards and criteria to which scientists themselves adhere.”

If we are to follow this path, then we must understand that science can never tell us what is and is not true: “It is scientific only to say what is more likely and what is less likely.”

**Notes**


2. All graphic illustrations were created utilizing ProStat Software.
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www.nacdl.org/costsavings
\[ \mu_c = \sqrt{s_1^2 + s_2^2 + \cdots + s_n^2} \]

16. Expanded uncertainty:

\[ U = k\mu_c \]

With this information, a complete result and coverage interval can be expressed as:

Result:

\[ X = y \pm U \ (95\%) = y \pm k\mu_c \ (95\%) \]

Coverage interval:


19. See note 15.

20. Example adapted from Gullberg, Estimating the Measurement Uncertainty in Forensic Breath Alcohol Analysis, 11 ACCRED. QUAL. ASSUR. 562, 562 (2006); Gullberg, Common Legal Challenges and Responses in Forensic Breath Alcohol Determination, 16(2) FORENS. SCI. REV. 92, 93 (2004).

21. Remember, a test result consists of both the test value and its associated uncertainty.


24. SWGDRUG, Recommendations, Part IV-C § 1.1 (5th ed. 2010).


27. NAS, STRENGTHENING FORENSIC SCIENCE IN THE UNITED STATES: A PATH FORWARD 184 (2009).


30. The determination is based upon the assumption that the underlying distribution is Gaussian (i.e., normal, a Bell Curve). Confidence Interval: \( 0.0731 \rightarrow 0.0877 \)

\[ \hat{y}_{bc} \text{ (bias corrected mean):} \]

\[ \frac{(b + b_2)}{2} = \frac{(0.0731 + 0.0877)}{2} = 0.0804 \]

\( U \) (expanded uncertainty — 99%):

\[ \hat{y}_{bc} - b_1 \leq 0.0804 - 0.0731 = 0.0073 \]

\( \mu_c \) (combined uncertainty):

\[ \frac{U}{2.576} \cdot \frac{0.0073}{2.576} = 0.00283 \]

\( z_{1.96} = 0.99 \) (z-factor):

\[ \frac{0.00283}{0.00283} = 1.41 \]

\[ P_{b<0.00} \] (probability BAC less than 0.08):

\[ \approx 44\% \ [z_{1.41} \rightarrow \text{Table}] \]

The use of this particular example should not be taken as an indication that the uncertainty can only impact the outcome at values very near a critical limit. The evidence showed that when uncertainty was included, results with mean values of 0.030 greater than a particular limit could actually be shown to include values below the limit in question. Moreover, these values were shown to be conservative so that even results in excess of these may be found to include values below a particular limit contained within their associated coverage interval.

31. King County District Court judges David Steiner, Darrell Phillipson and Mark Chow.


34. The only testimony was that of University of Washington Metrologist Dr. Ashley Emery.

35. The state’s experts were the former head of the Washington State Breath Test Section Rod Gullberg, State Toxicologist Fiona Couper, and State Toxicology Lab Quality Assurance Manager Jason Sklerov.

36. The King County Prosecutor’s Office is headed by prosecutor Dan Satterberg.

37. NAS, STRENGTHENING FORENSIC SCIENCE IN THE UNITED STATES: A PATH FORWARD 186 (2009).


40. Id. at 28.

41. Id. at 17.

42. Id. at 26.

43. Id. at 28-29.

44. Commonwealth of Northern Mariana Islands v. Bowie, 243 F.3d 1109, 1114 (9th Cir. 2001).


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Author’s Bio

TED VOSK was kicked out of his house as a teenager and lived on the streets, homeless, for the better part of four years. Eventually tiring of his situation, he gained admittance to Eastern Michigan University where he earned the status of national Goldwater Scholar before graduating with honors in Theoretical Physics and Mathematics. Thereafter Ted entered the Ph.D. program in Physics at Cornell University before moving on to Harvard Law School where he obtained his JD. Ted is currently a legal and science writer, criminal defense attorney, and legal/forensic consultant. A member of the American Academy of Forensic Sciences and Mensa, he has written, broadcast, presented and taught around the country on topics ranging from string theory to the doctrine of constitutional separation of powers. He has been part of the most significant DUI cases in Washington State over the past decade. For this work, he has received the President’s Award from the Washington Association of Criminal Defense Lawyers and the Certificate of Distinction from the Washington Foundation for Criminal Justice. He has been published in multiple legal periodicals and treatises on DUI law and is currently authoring a textbook on Forensic Metrology. Ted lives in Washington with his wife Kris. Whether it’s chasing down active volcanoes, traveling with friends or simply sharing a sunset in the mountains, the two of them strive to suck every last ounce of marrow out of life. Quiet moments are often spent with their dog and cats simply enjoying the wonder of it all.

4 BIPM, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM), § 5.2 (2008); BIPM, International Vocabulary of Metrology — Basic and General Concepts and Associated Terms (VIM), § 0.1 (2008).
8 Desimoni, About considering both false negative and false-positive errors when assessing compliance and non-compliance with reference values given in compositional specifications and statutory limits, 13 ACCRED. QUAL. ASSUR. 653, 653 (2008); BIPM, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM), § 0.1 (2008); Richter, Reporting measurement uncertainty in chemical analysis, 13 ACCRED. QUAL. ASSUR. 113, 113 (2008); Watson, Pharmaceutical Analysis - A Textbook for Pharmacy Students and Pharmaceutical Chemists, 2 (2nd ed. Elsevier 2005).


14 NAS, 117.

15 Bich, Interdependence between measurement uncertainty and metrological traceability ACCRED. QUAL. ASSUR. (IN PRESS - 2009).

16 All graphic illustrations were created utilizing ProStat Software.

17 Feynman 165-166 (1965).


20 GUM § 0.1 (2008).


22 GUM § 3.2.2 n.2 (2008).


25 GUM § 2.3.5 (2008).


27 GUM § 2.36 (2008).

28 NIST 1297 § 2.1 (1994); GUM § 3.1.2 (2008).

29 EURACHEM QUAM p.1, § 1.0 (2000).


32 Kacker 517 (2007).

33 See also, Vosk, Computational Aspects of Measurement Uncertainty in Washington State Breath Alcohol Tests, Wash. Prac., DUI Practice Manual Appendix (Callahan ed., In Press); Vosk, Trial by Numbers: Uncertainty in the Quest for Truth and Justice 56 THE NACDL CHAMPION 54, Fn. 30 (Nov. 2010)(reprinted with permission in 40(3) THE VOICE FOR THE DEFENSE 31 (April 2011)). Determining the z-factor assumes a Gaussian distribution. The same method can be used with a t-distribution as long as we know the initial value of t utilized to create the expanded uncertainty.

34 See also, Vosk, Computational Aspects of Measurement Uncertainty in Washington State Breath Alcohol Tests, Wash. Prac., DUI Practice Manual Appendix (Callahan ed., In Press); Vosk, Uncertainty in Forensic Breath Alcohol Testing, Intoxication Test Evidence, Ch. 56 (E. Fitzgerald ed., 2nd ed. 2009); Vosk, Using the database and QAP to determine the likelihood that a test result greater than a .08 reflects a true BAC less than the per se limit, § 11.7 Defending DUI’s in Washington – 2008 Update (Cowan & Fox, 3rd ed. 2007).

35 Bono 7 (2009).


41 JCGM, International Vocabulary of Metrology — Basic and General Concepts and Associated Terms (VIM), § 2.3 (2008).

42 JCGM, International Vocabulary of Metrology — Basic and General Concepts and Associated Terms (VIM), § 0.1 (2008).


44 BIPM, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM), § 3.2.4 (2008); NIST, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST TN 1297 § 5.2 (1994).
The example assumes that the uncertainties are independent.

This example is taken from Gullberg, *Estimating the uncertainty associated with Widmark’s equation as commonly applied in forensic toxicology* 172 FOR. SCI. INT. 33 (2007), and fills in some blanks not explicitly shown therein.

Assuming only \( \tau \) and \( \beta \) are not independent.

Assuming only \( \tau \) and \( \beta \) are not independent.